ME 301
MECHANICS OF MACHINERY

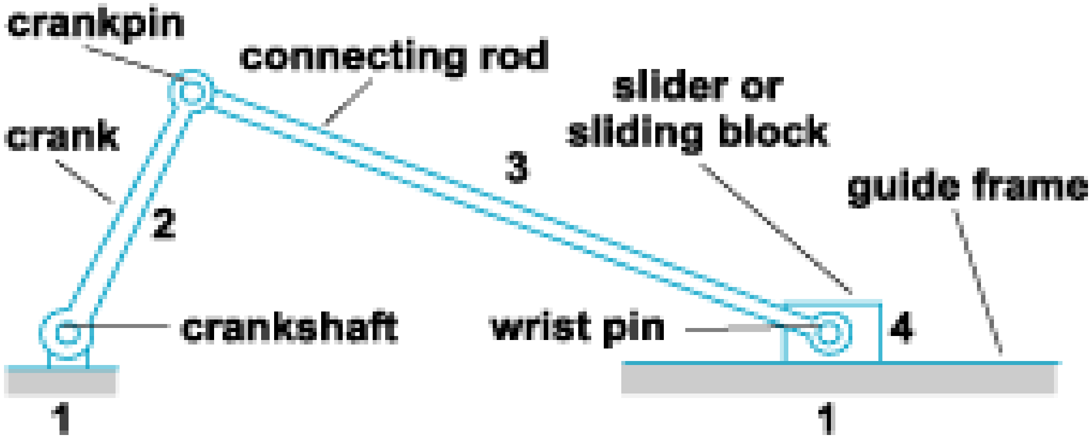
MODULE 1

INTRODUCTION

Mechanism

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as mechanism

Eg: A slider crank mechanism



MACHINE

A machine is a mechanism or a combination of mechanism, a device for transferring and transforming motion and force(power) from source to load.

Eg: Automobile engine

Difference between machine & mechanism

Machine	Mechanism
1. Machine modifies mechanical work	1. Mechanism transmits and modifies motion
2. A machine is a practical development of any mechanism	2. A mechanism is a part of a machine
3. A machine may have number of mechanism for transmitting mechanical work or power	3. A mechanism is a skeleton outline of the machine to produce motion between various links.

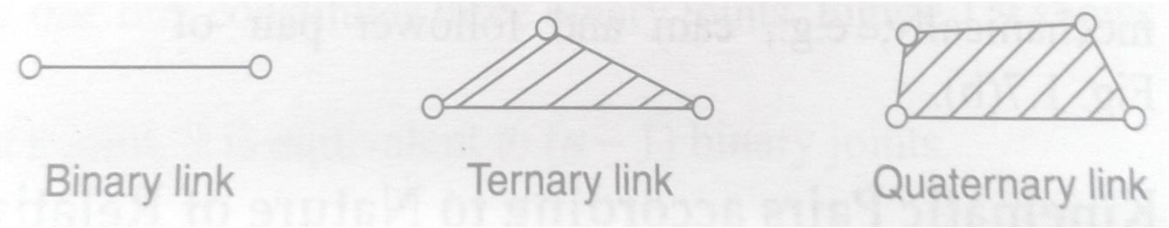
RIGID AND RESISTANT BODIES

A body is said to be rigid if it under the action of forces, it does not suffer any distortion.
Resistant bodies are those which are rigid for the purposes they have to serve.

Examples:
A belt is rigid when subjected to tensile forces. Therefore belt drive act as a resistant body.
Fluids can also act as resistant bodies when compressed as in case of a hydraulic press.

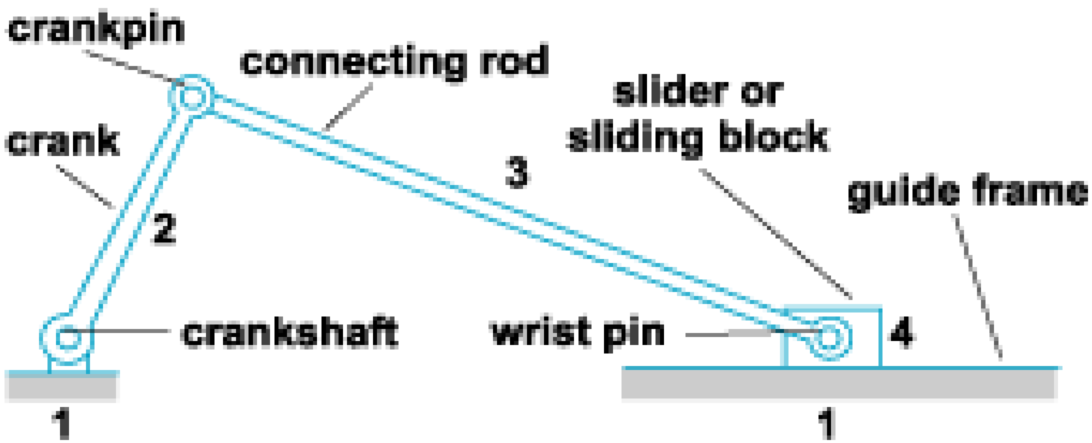
LINKS

- A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a link.
- It can be also defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them
- Links can be classified into binary, ternary and quaternary depending upon their ends on which revolute or turning pairs can be placed



KINEMATIC PAIR

A kinematic pair or simply a pair is a joint of two links having relative motion between them.



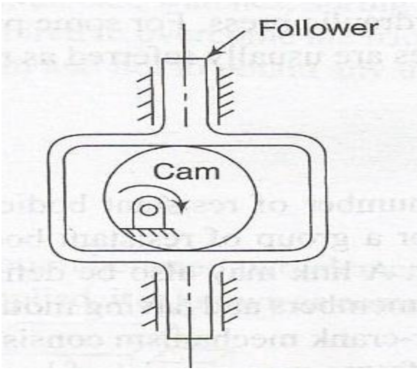
Link 1&2, 2&3, 3&4--turning pair or revolute
Link 1&4-- Sliding pair

TYPES OF KINEMATIC PAIRS

According to the nature of mechanical constraint

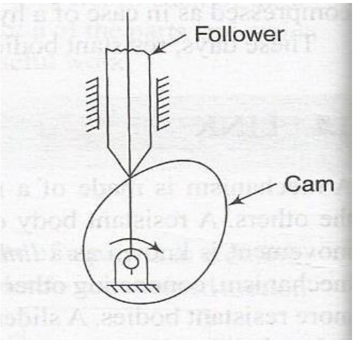
a) Closed pair

- Elements of pair are held together mechanically



b) Unclosed pair

- Links are not held together mechanically
- Two links of a pair are in contact either due to force of gravity or some spring action



According to the nature of contact

a) Lower pair

A pair of links having surface or area contact between the members

Eg:- nut turning on a screw, shaft rotating in a bearing, all pairs of slider crank mechanism

b) Higher pair

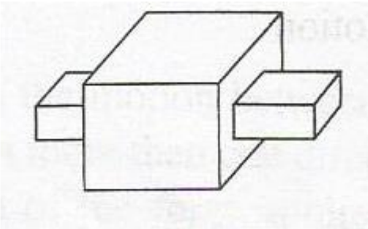
A pair of links having a point or line contact is known as higher pair

Eg:- wheel rolling on the surface, cam and follower, meshed gears, ball and roller bearing

According to the nature of relative motion

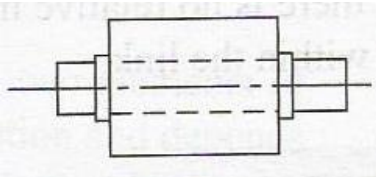
a) Sliding Pair

Here two links have a sliding motion relative to each other. In the figure A can only slide in B.



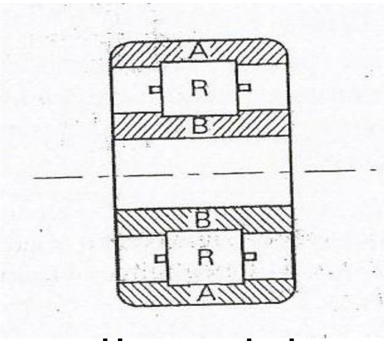
b) Turning pair

when one link has a turning or revolving motion relative to other. In the figure A can only rotate in B



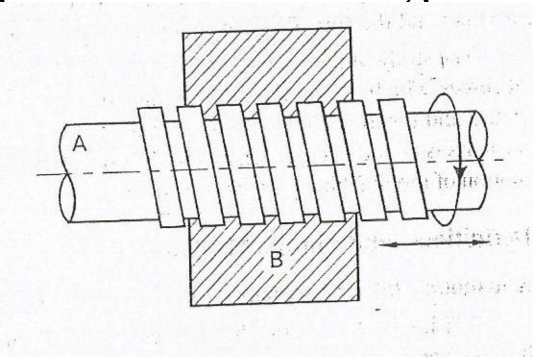
c) Rolling pair

when the links of a pair have a rolling motion relative to each other, they form a rolling pair. In the figure below the rollers R are rolling on the surfaces of inner race B and the other race A.



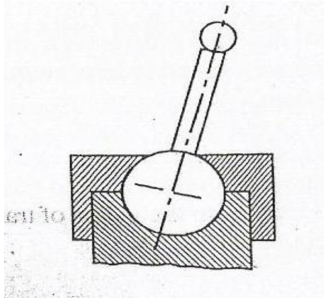
d) Screw Pair

If the two mating links have a turning as well as sliding motion between them, they form a screw pair.



e) Spherical Pair

when one link in the form of a sphere turns inside a fixed link, it is a spherical pair.



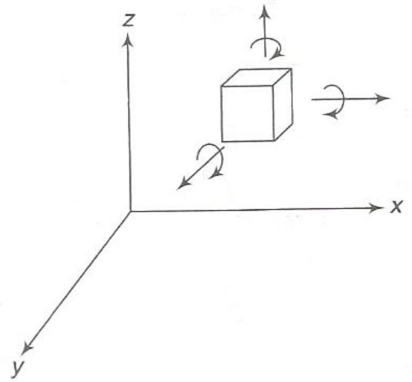
Degrees of Freedom

Degree of freedom of a pair is defined as the number of independent relative motion, both translational and rotational, a pair can have.

DOF= (6 - no. of constraints)

An unconstrained rigid body moving in space can describe the following independent motions:

1. Translational motion along any three mutually perpendicular axis x, y, z
2. Rotational motion about these axes

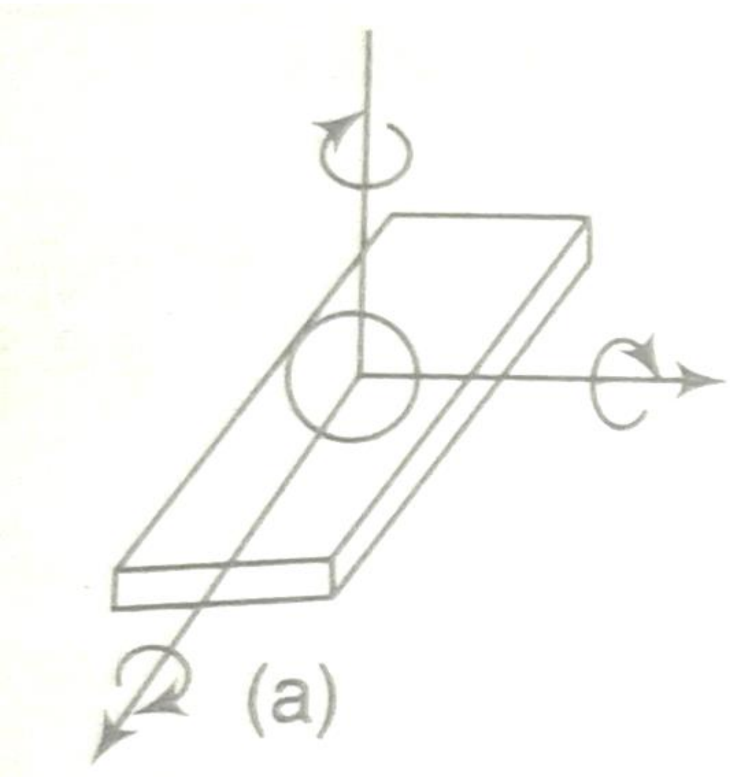


- The connection of link with another imposes certain constraints on their relative motion.
- The number of restraints can never be zero or six

Classification of kinematic pairs

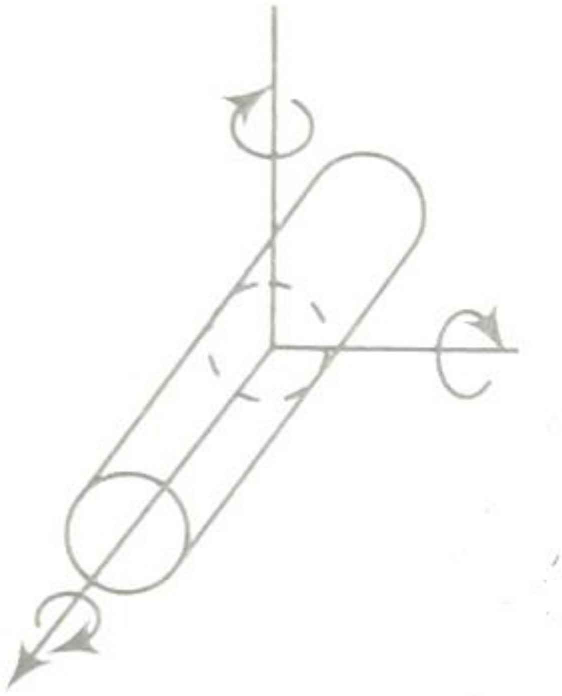
a) Sphere- plane

No. of constraints= 1



constraints on	
Translatory motion= 1	Rotory motion=0

b) Sphere- cylinder

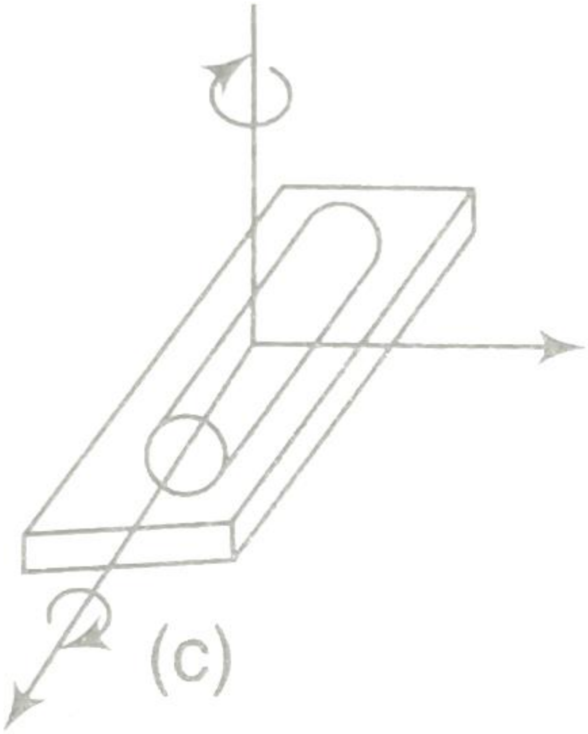


No. of constraints= 2

constraints on	
Translatory motion= 2	Rotory motion=0

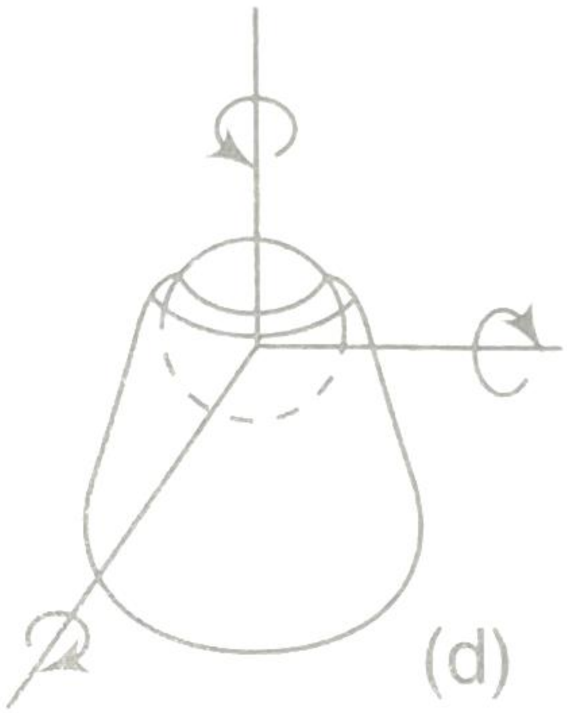
c) Cylinder- plane

No. of constraints= 2



constraints on	
Translatory motion= 1	Rotory motion=1

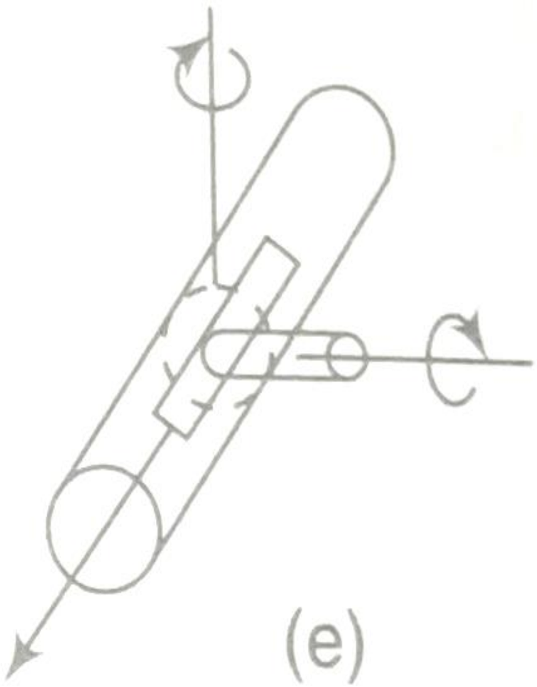
d) Spheric



No. of constraints= 3

constraints on	
Translatory motion= 3	Rotory motion=0

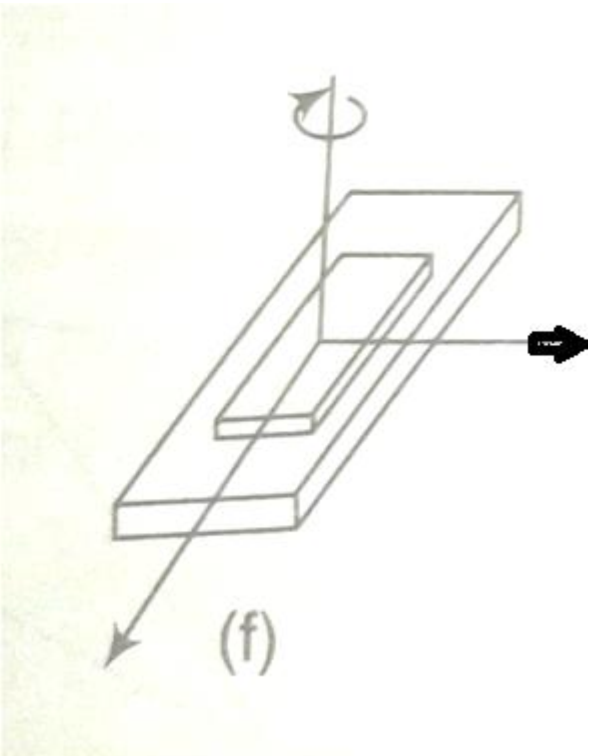
e) Sphere- slotted cylinder



No. of constraints= 3

constraints on	
Translatory motion= 2	Rotory motion=1

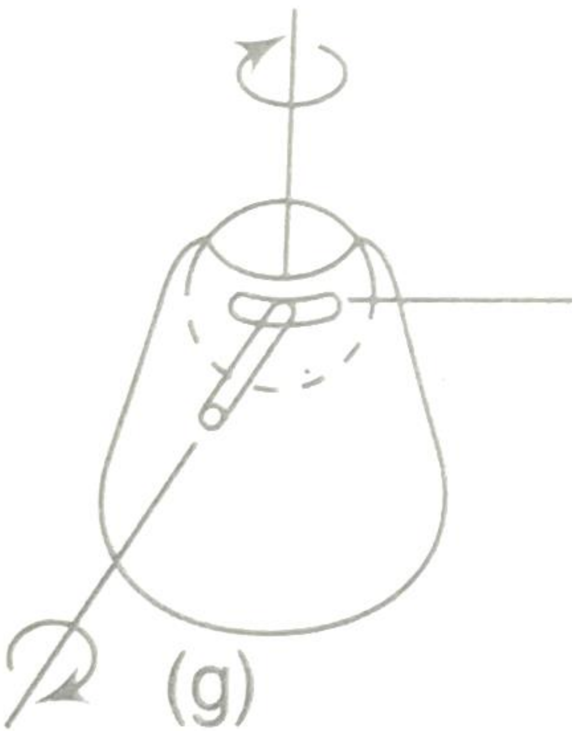
f) Prism- plane



No. of constraints=3

constraints on	
Translatory motion= 1	Rotory motion=2

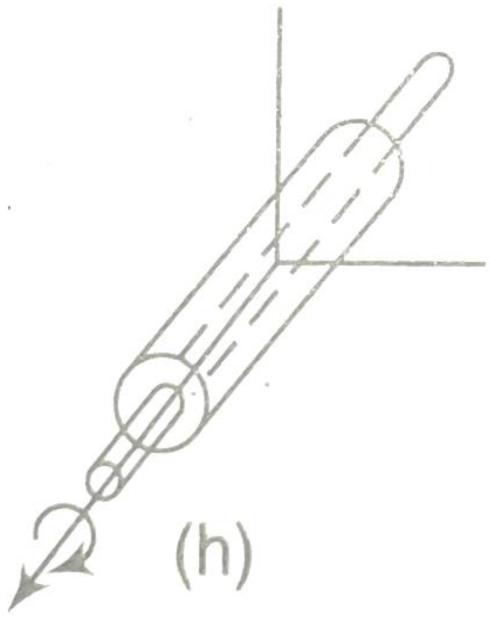
g) Slotted- spheric



No. of constraints=4

constraints on	
Translatory motion= 3	Rotory motion=1

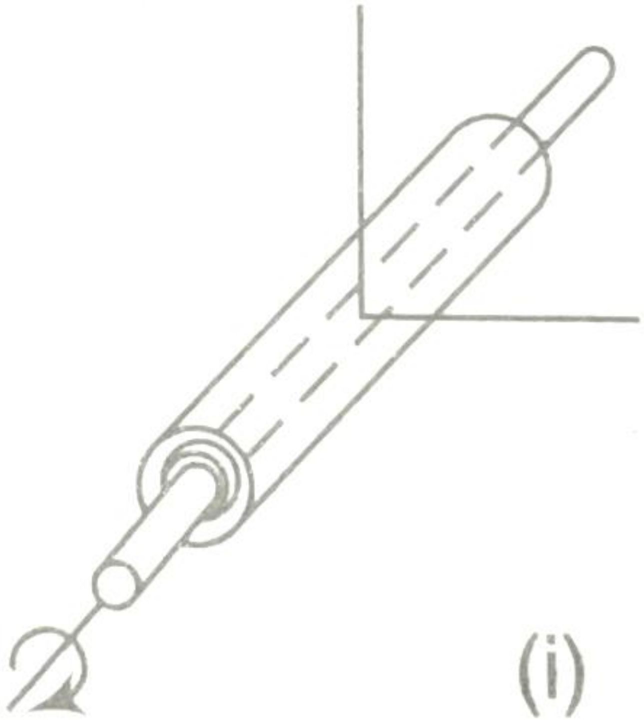
h) Cylinder



No. of restraints=4

constraints on	
Translatory motion= 2	Rotory motion=2

i) Cylinder -collared

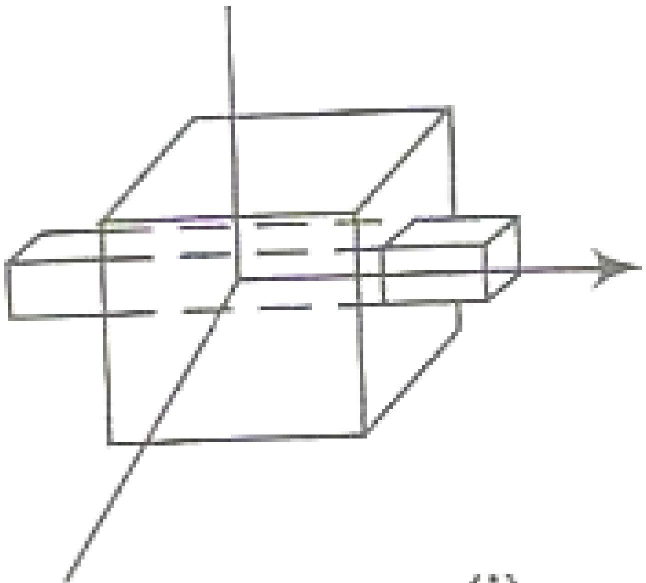


(i)

No. of constraints= 5

constraints on	
Translatory motion= 3	Rotory motion=2

j) Prismatic


(j)

No. of restraints=5

constraints on	
Translatory motion= 2	Rotory motion=3

Mobility of mechanism

Mobility of a mechanism defines the number of degrees of freedom a mechanism posses.

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5$$

<u>GRUEBLER'S CRITERION</u>

$$F = 3(N-1) - 2P_1 - 1P_2$$

Where N= total no. of links in a mechanism

N-1= no. of movable links

P1 = no. of pairs having one degree of freedom

P2= no. of pairs having 2 dofs

Kutzback’s criterion

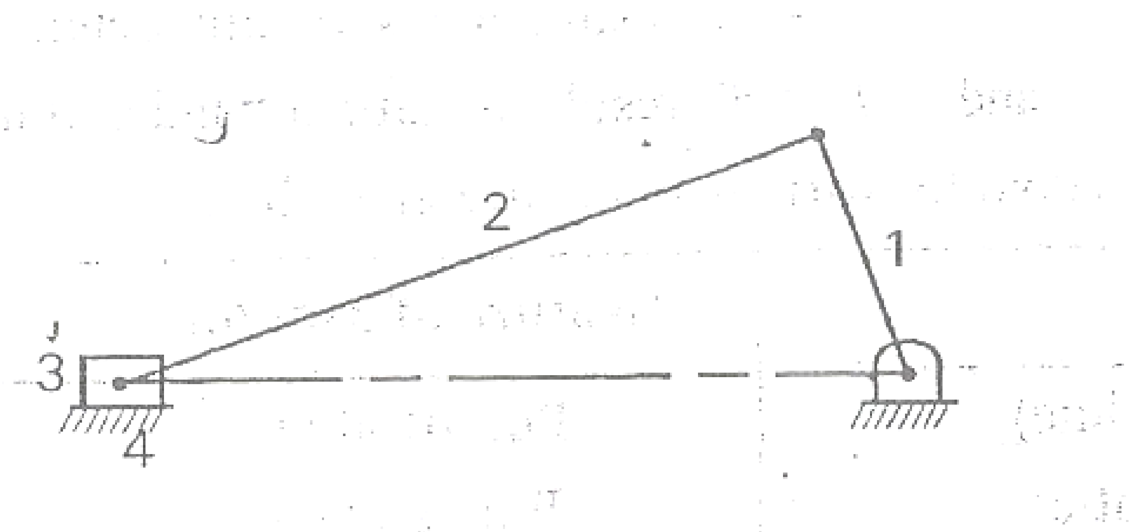
Some authors mention the Gruebler’s relation as kutzback’s criterion and a simplified relation :

$$F = 3(N-1) - 2P_1$$

This is applicable to linkages with single dofs , ie $P_2=0$

Degrees of freedom for various joints

Type of joint	**Nature of motion**	**DOF**
Hinges(Revolute)	Turning	1
Slider	Pure sliding	1
Cylindrical, cam , gear, ball bearing	Turning and sliding Turning and rolling	2
Rolling contact	Pure rolling	1
Spheric		3



N=4

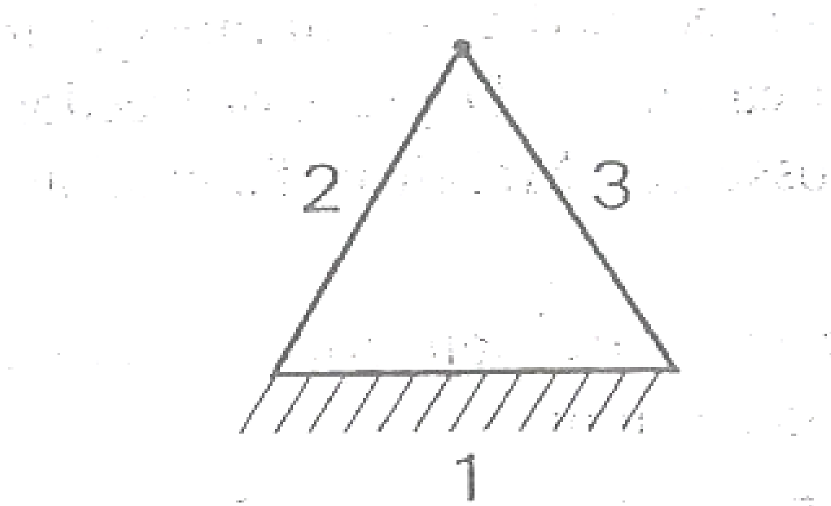
(N-1)= 3

P1=4

P2=0

$F=3(N-1)-2P_1-1P_2 \quad = \quad 3(3)-2(4)-1(0)=1$

Therefore the mechanism has 1 DOF



N=3

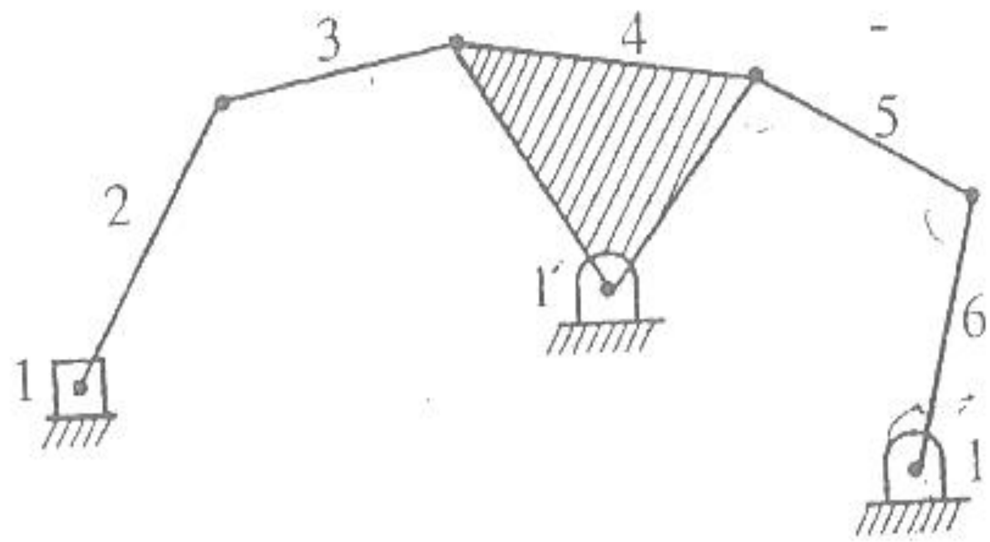
(N-1)= 2

P1=3

P2=0

$F=3(N-1)- 2P_1-1P_2 \quad = \quad 3(2)- 2(3)-1(0)= 0$

0 DOF = statically determinate structure



N=6

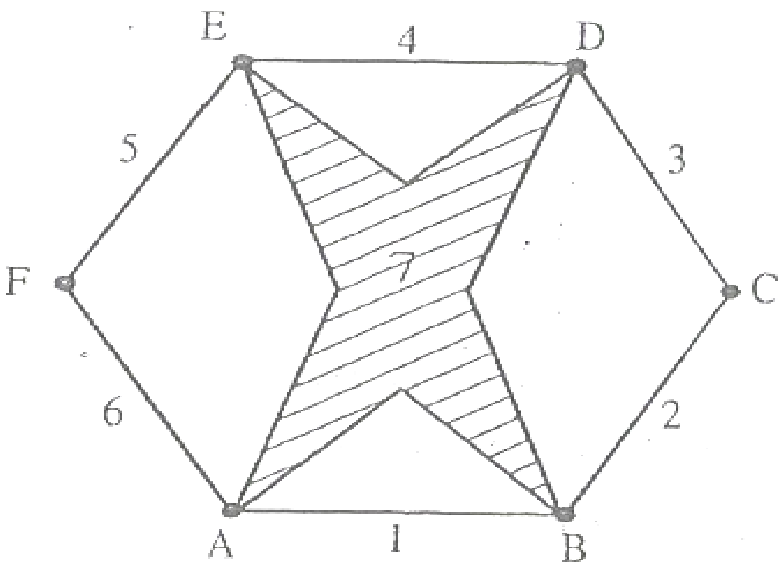
(N-1)= 5

P1=7

P2=0

$F=3(N-1)-2P_1-1P_2 \quad = \quad 3(5)-2(7)-1(0)=1$

Therefore the mechanism has 1 DOF



N=7

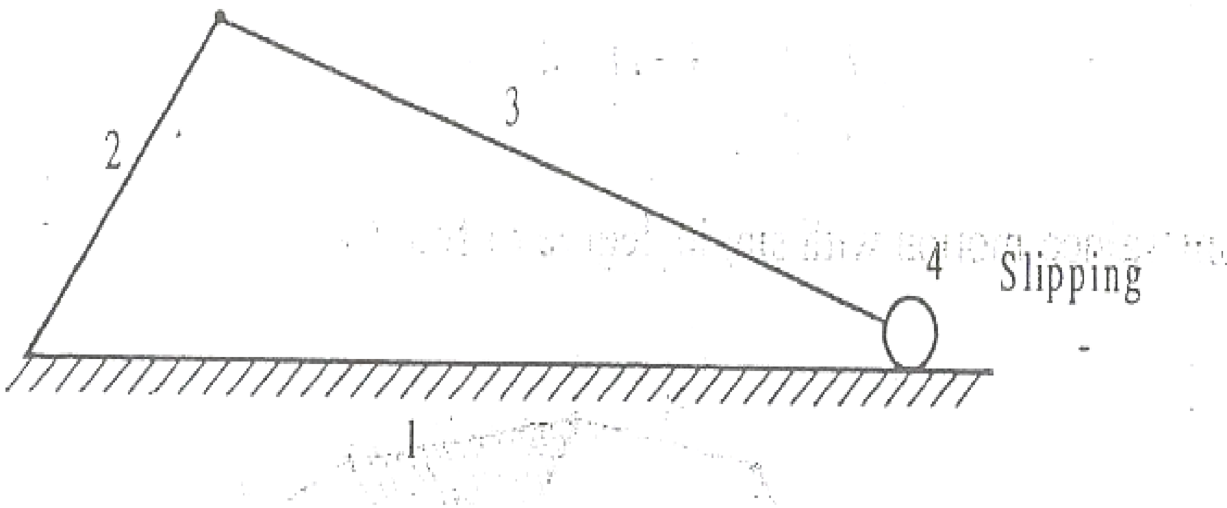
(N-1)= 6

P1=10

P2=0

$F=3(N-1)-2P_1-1P_2 = 3(6)-2(10)-1(0)=-2$

Hence it is statically indeterminate structure



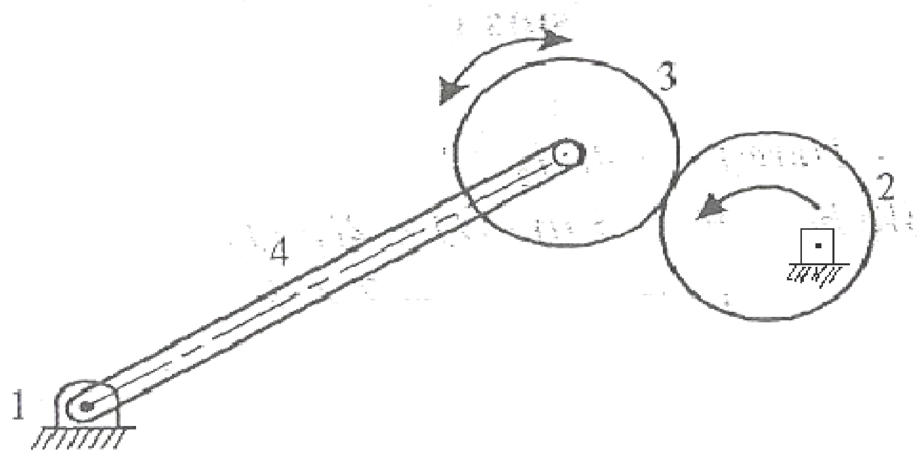
N=4
(N-1)= 3
P1=3
P2=1 (rolling and sliding)
$F=3(N-1)- 2P_1-1P_2 \quad = \quad 3(3)- 2(3)-1(1)= 2$

Therefore the mechanism has 1 DOF

Redundant degrees of freedom

Sometimes one or more links of a mechanism can be moved without causing any motion to the rest of the links of the mechanism. Such a link is said to have redundant dof. So it is necessary to find such links prior to investigate the dof of the whole mechanism.

EXAMPLE:

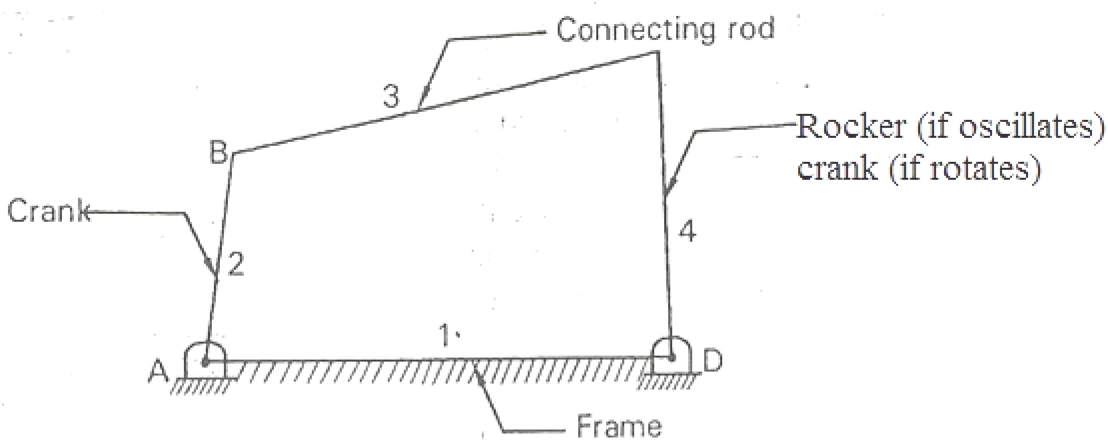


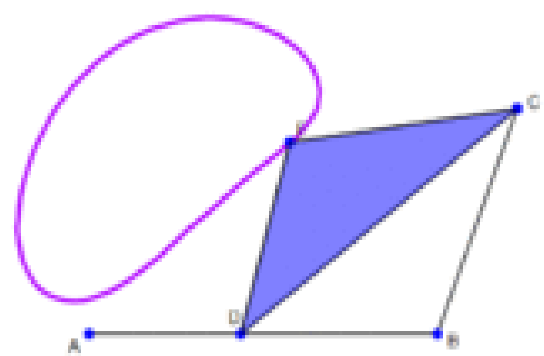
In the figure roller 3 can rotate about its axis without causing any movement to the rest of the mechanism

$F = 3(N-1) - 2P_1 - 1P_2 - F_r$ F_r = is the number of redundant dof

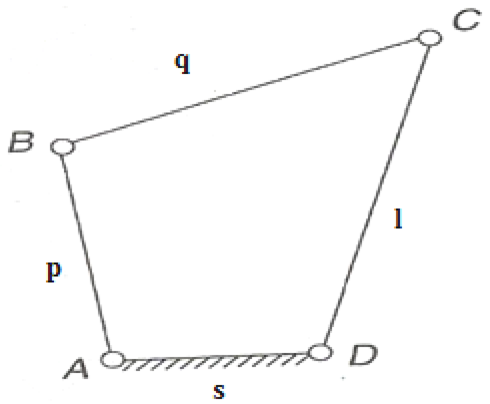
Therefore $F = 3(3) - 2(3) - 1(1) - 1 = 1$

Four bar chain or quadric cycle chain





Four bar chain or quadric cycle chain



In the above figure necessary conditions for the double crank is:

- Link s– the shortest link should be fixed
- The sum of the length of shortest (s) and longest (l) links is less than the sum of the other two link

ie, in the above fig:

Link (s + l) < (p + q)

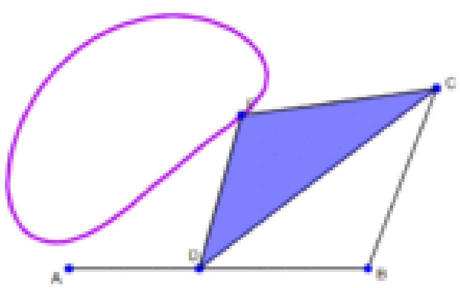
The mechanism thus obtained is known as crank-crank or double- crank mechanism

Inversions of four bar mechanism

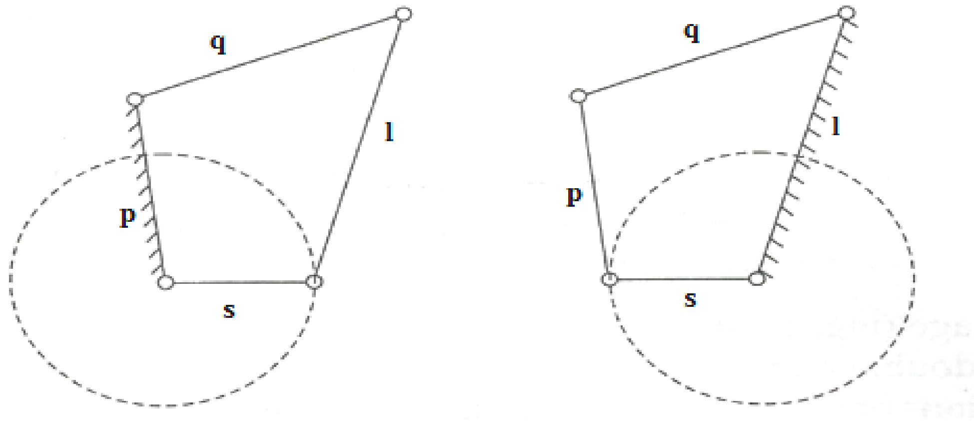
First inversion: crank- lever mechanism or crank- rocker mechanism

- If any of the adjacent links of link s, i.e either p or l is fixed, s can have a full revolution and the link opposite to it oscillates (rocks).

- Link **(s + l) < (p + q)**

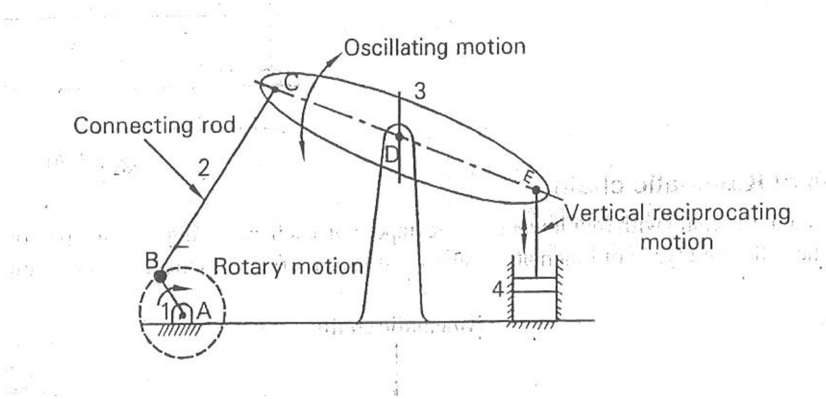


In fig (a): p is fixed , s crank and q oscillates
In fig (b): l is fixed, s crank and q oscillates



(a) (b)

Application : Beam engine

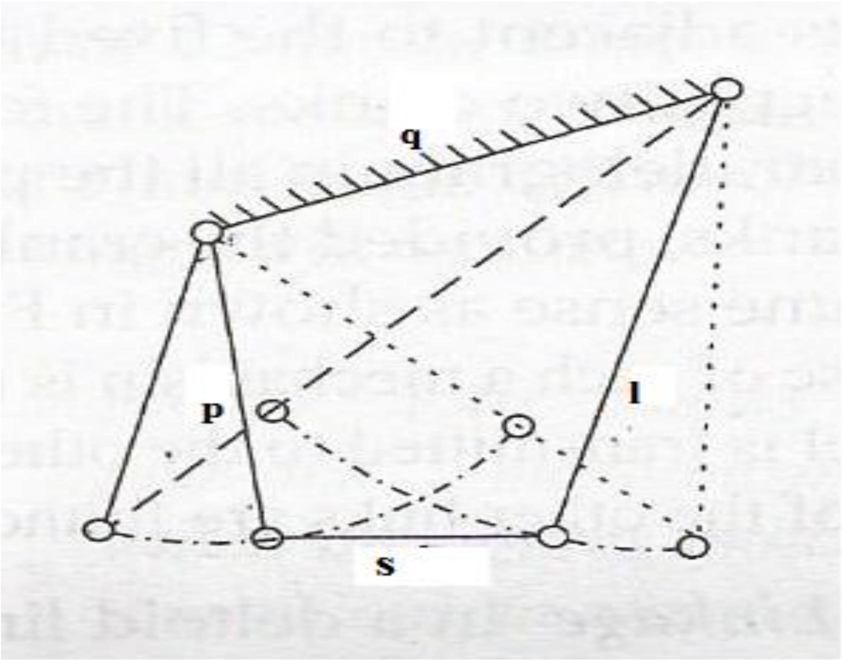


Oil pump jack

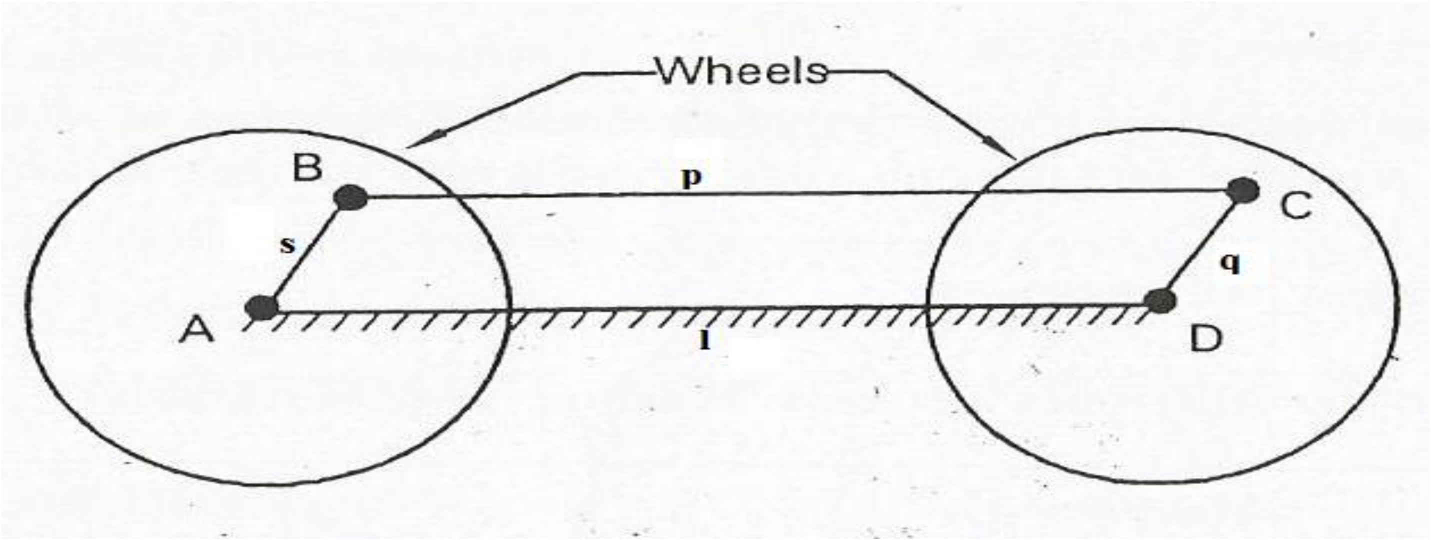
Second inversion: Double lever or double rocker mechanism

•If the link opposite to the shortest link, i.e q is fixed and the shortest link s is made a coupler, the other two links l and p would oscillate.

•Link **(s + l) > (p + q)**



Third Inversion: Double crank mechanism (coupled wheels of locomotive)



- In this mechanism the **length of link p= length of link l**
- **Links (s + l) = (p + q)**
- The two links adjacent to fixed link act as cranks
- Other wise called as parallel crank four bar linkage

Grashof's law

- Grashof's law states that a four bar mechanism has atleast one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of other two links

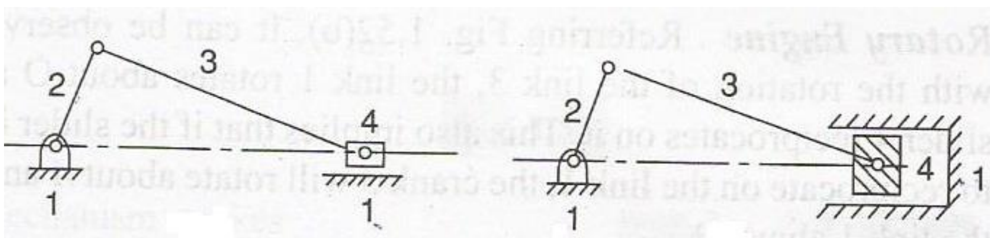
- If the link adjacent to the fixed link (adjacent to the shortest link) is fixed, the chain will act as crank rocker mechanism.

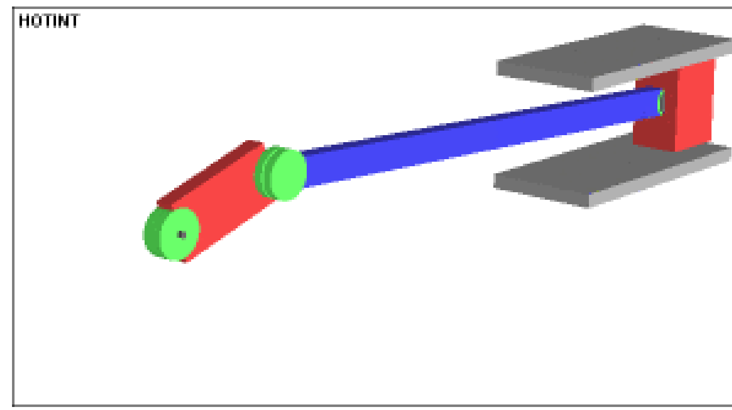
- If the link opposite to the shortest link is fixed, the chain will act as double rocker mechanism, in which links adjacent to the fixed link oscillates

- If the shortest link is fixed the chain will act as a double crank mechanism in which links adjacent to the fixed link have complete revolution

Slider –Crank chain

When one of the turning pairs of a four- bar chain is replaced by a sliding pair, it becomes a slider- crank mechanism



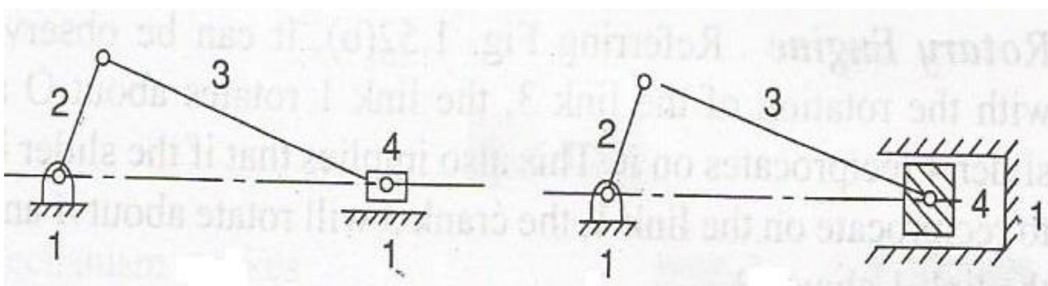


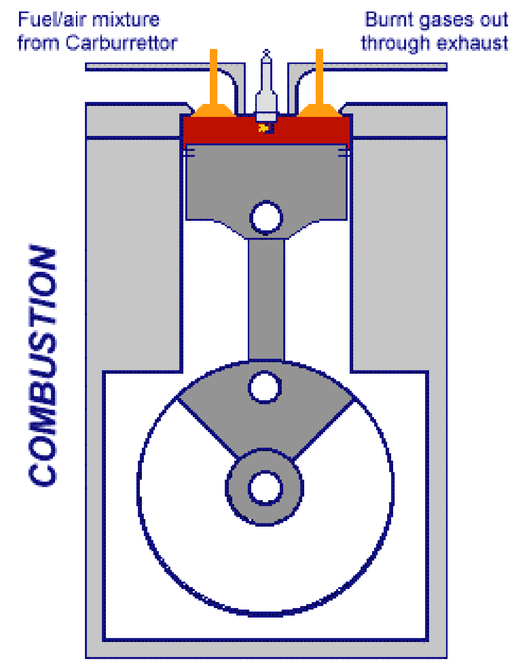
Sometimes the straight line path of the slider may be passing through the fixed pivot or may be displaced. If displaced the distance between the fixed pivot and the straight line path of the slider is called offset and the chain is called **offset slider crank chain**

Inversions of slider crank chain

First inversion

Link 1 is fixed and link 2and 4 are made the crank and slider

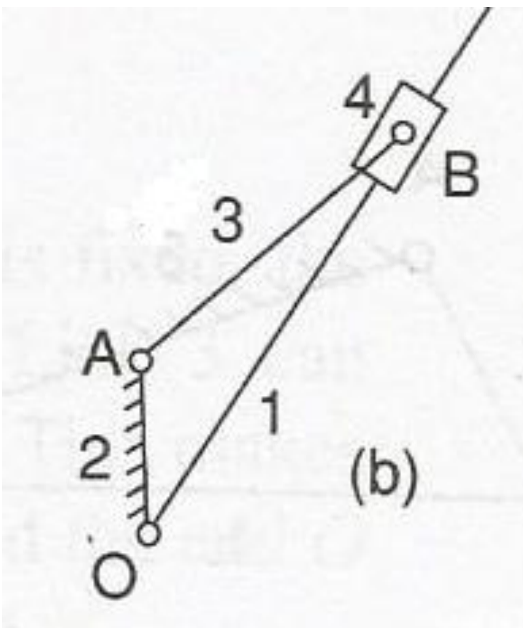


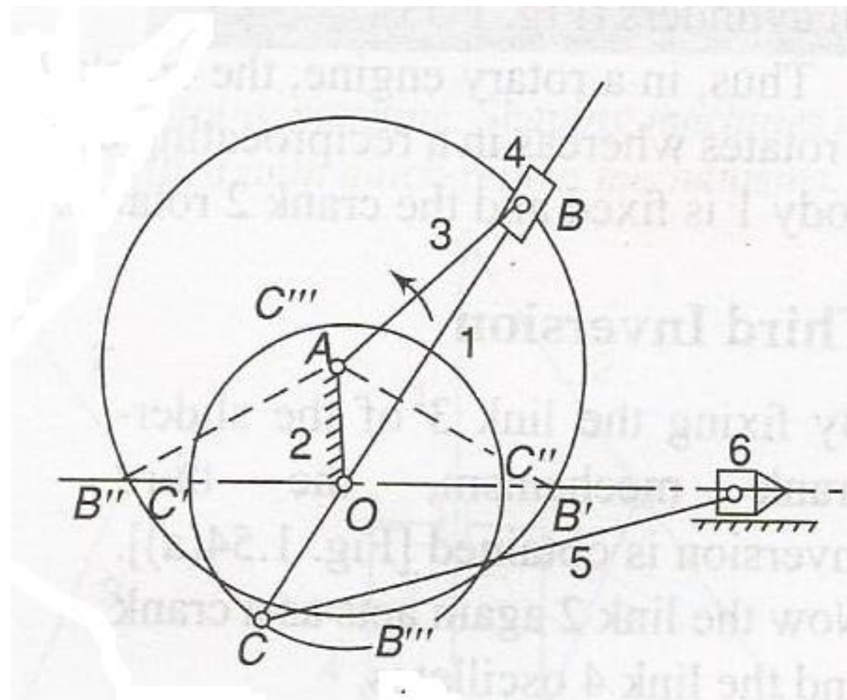


Applications: reciprocating engine, reciprocating compressor

Second Inversion

Link 2 is fixed for obtaining second inversion

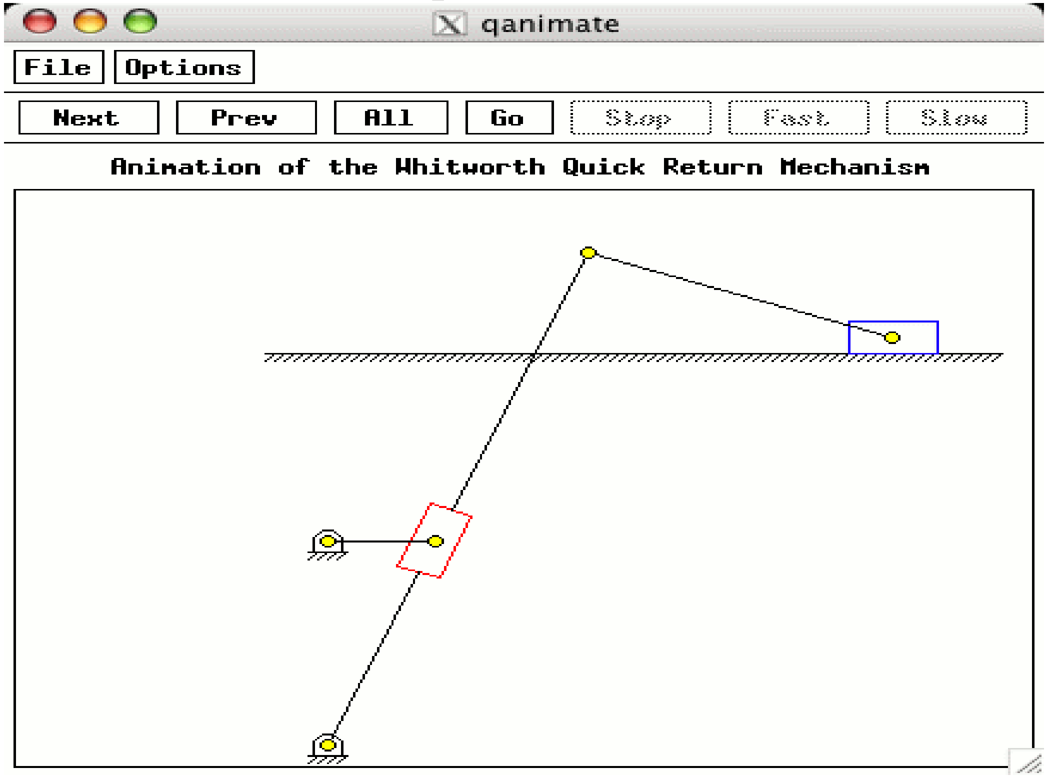




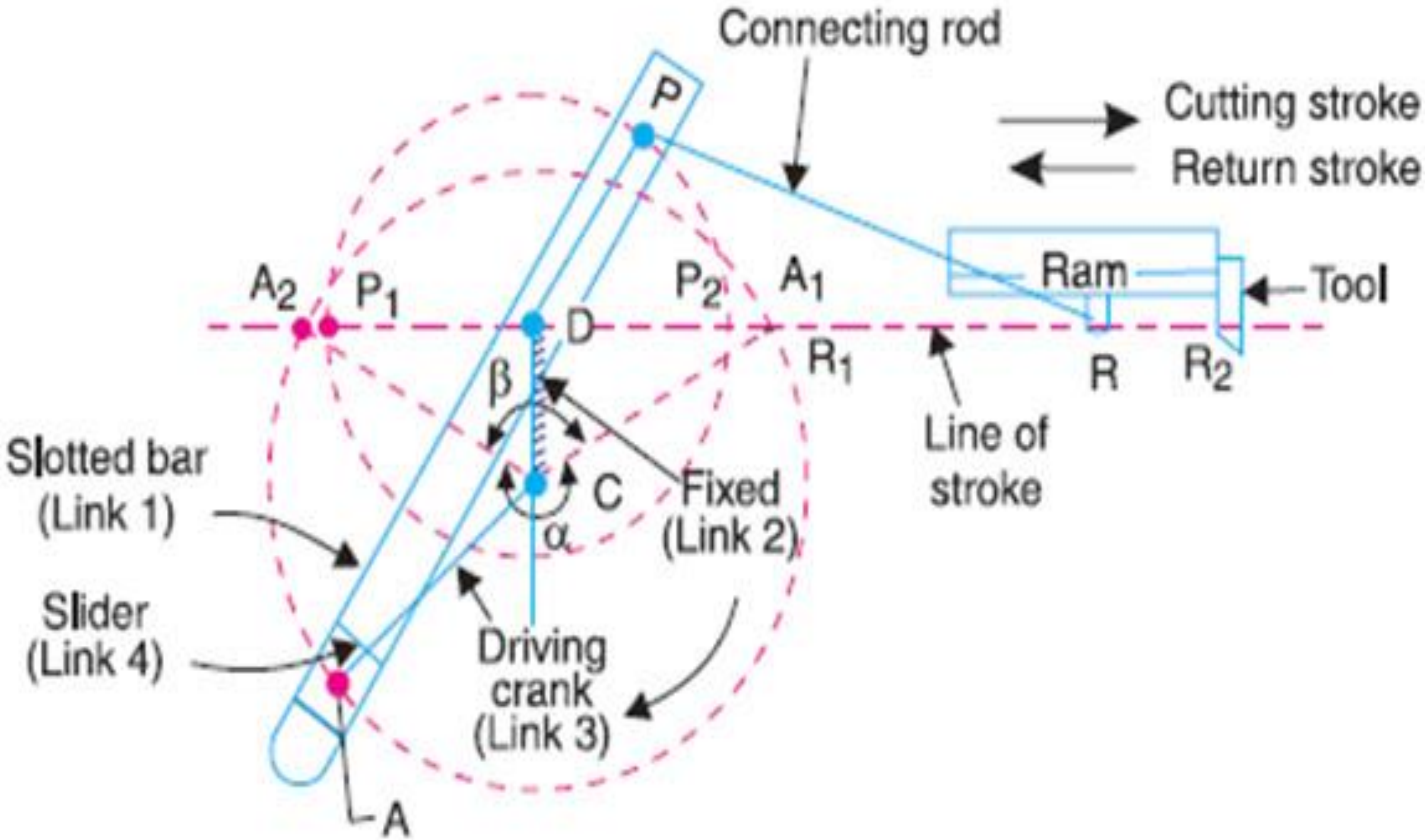
Application : whitworth quick- return mechanism, rotary engine

Whitworth quick- return mechanism

- Used in workshops to cut metals
- The forward stroke takes a little longer and cuts the metal, whereas the return stroke is idle and takes a shorter period

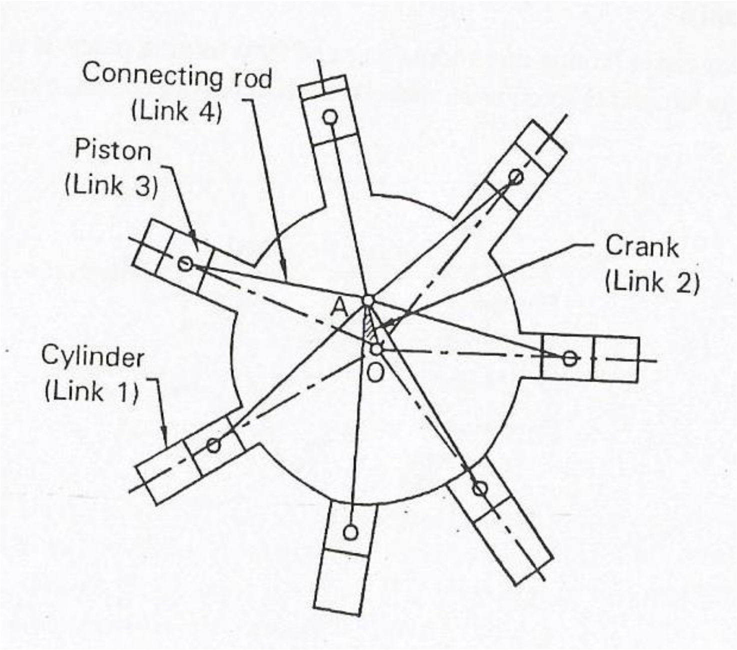


Whitworth quick- return mechanism



Rotary Engine

•In rotary engines the crank 2 is fixed and the body 1 rotates

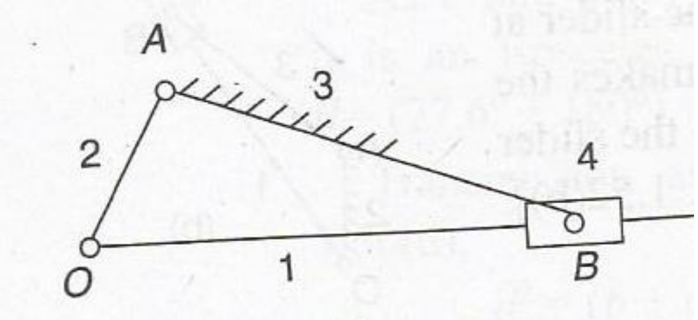


•In this engine it can be observed that with the rotation of link 3, the link 1 rotates about O and the slider reciprocates on it

Third Inversions

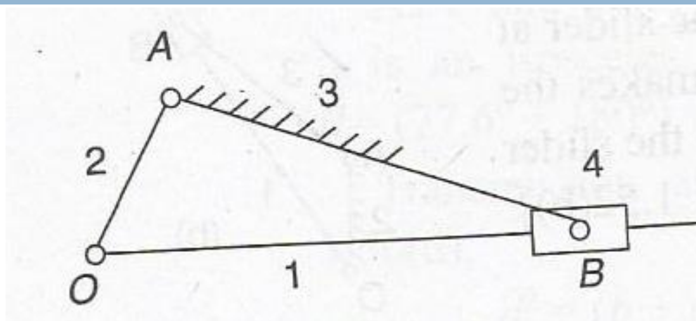
•Obtained by fixing the link3 of the slider crank mechanism

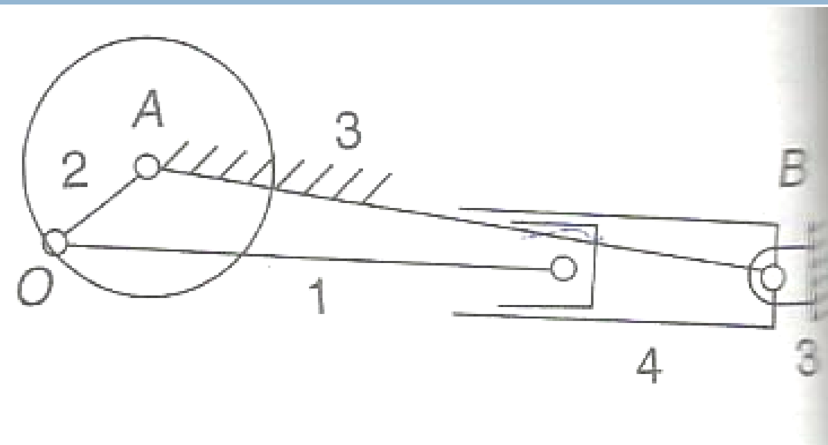
•Link 2 act as crank, link 4 oscillates

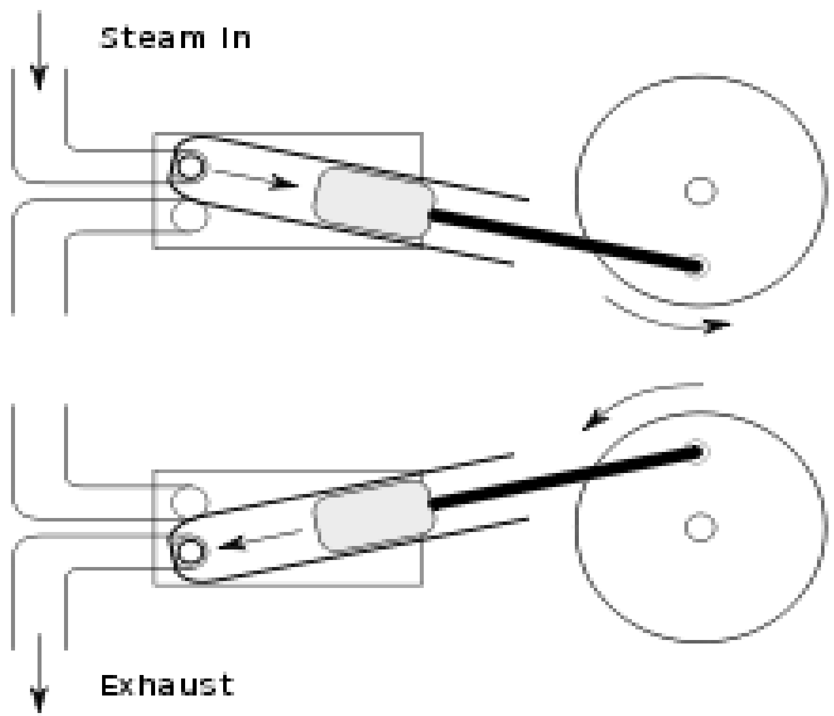


Applications: Oscillating cylinder engines, crank and slotted lever mechanism

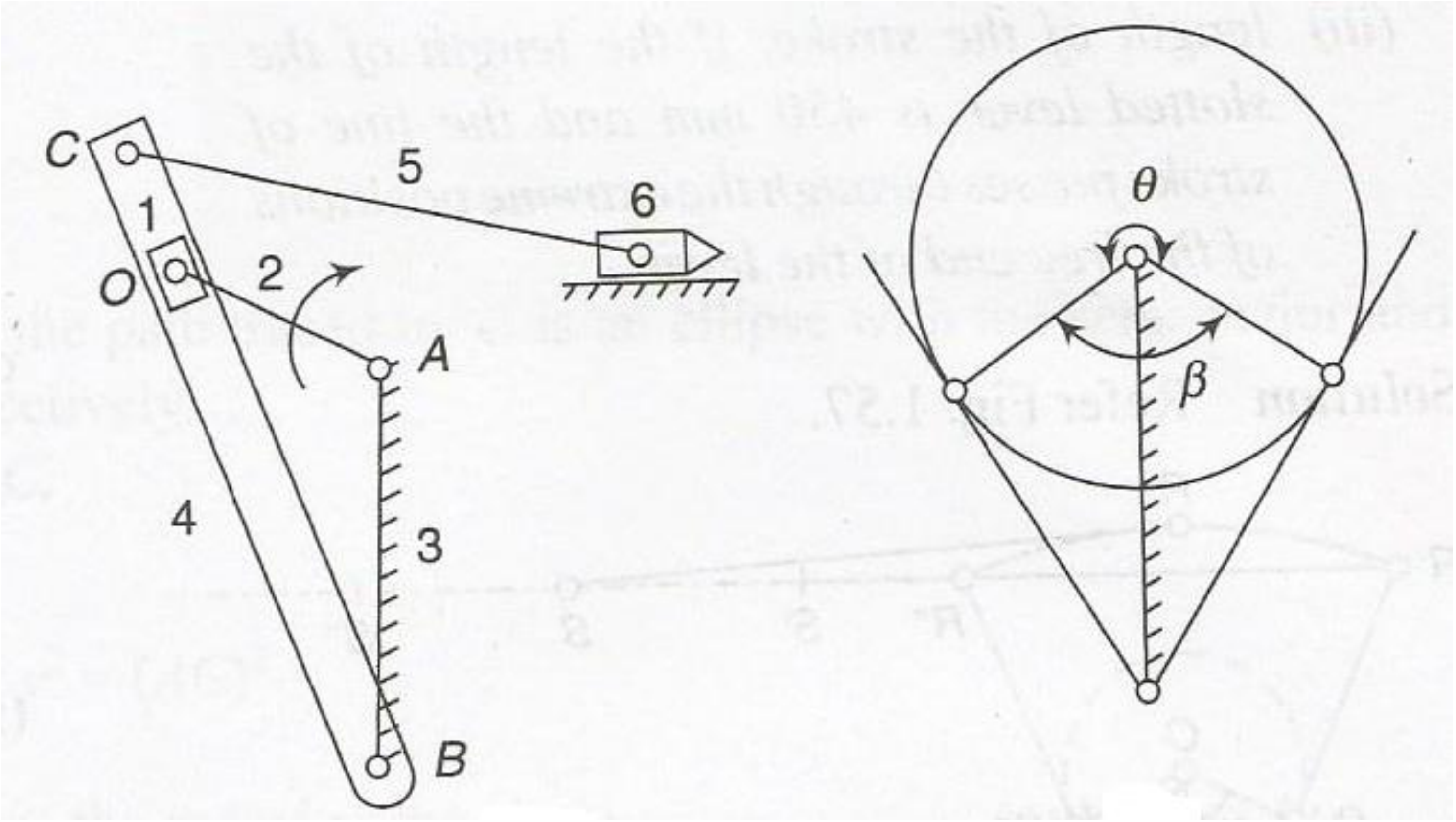
Oscillating cylinder engine



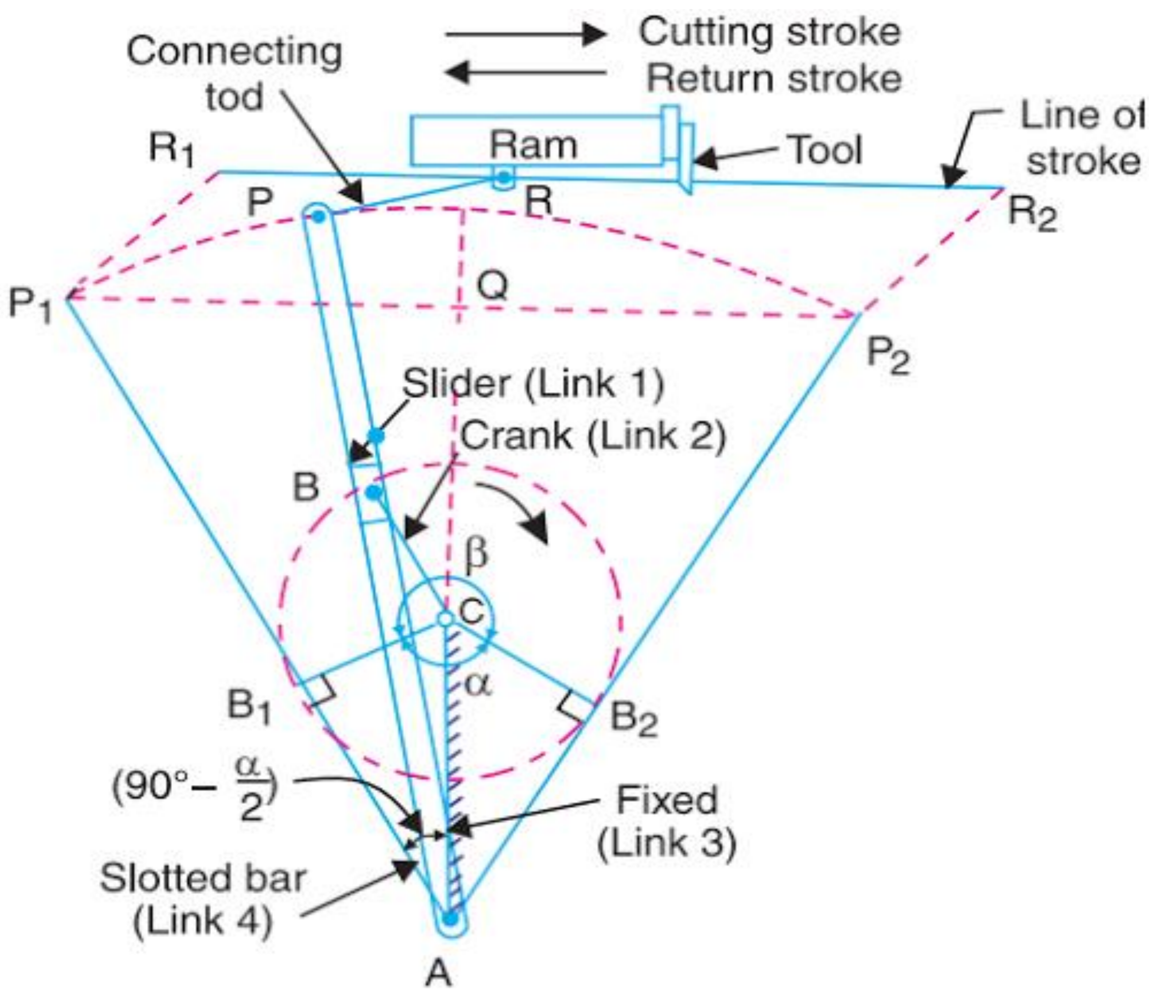




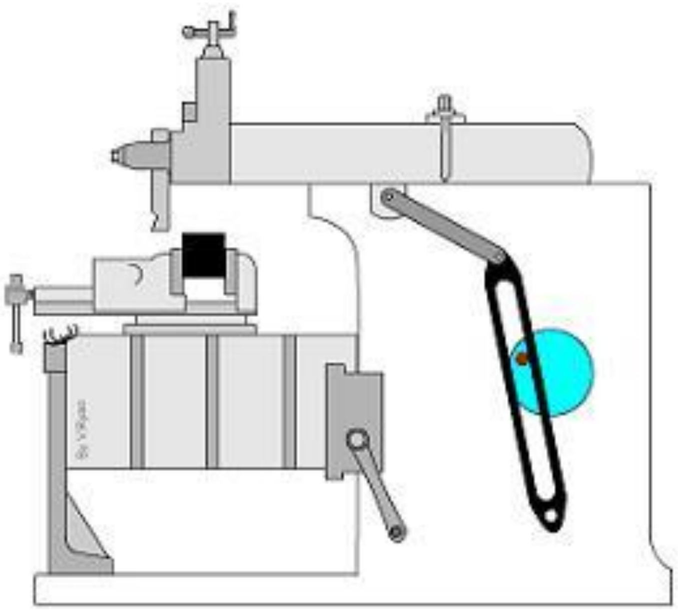
Crank & slotted lever quick return mechanism

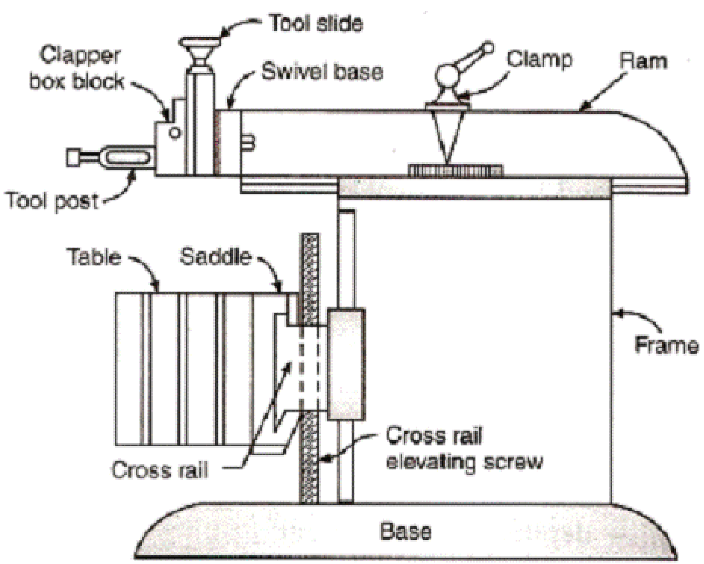


Crank & slotted lever quick return mechanism

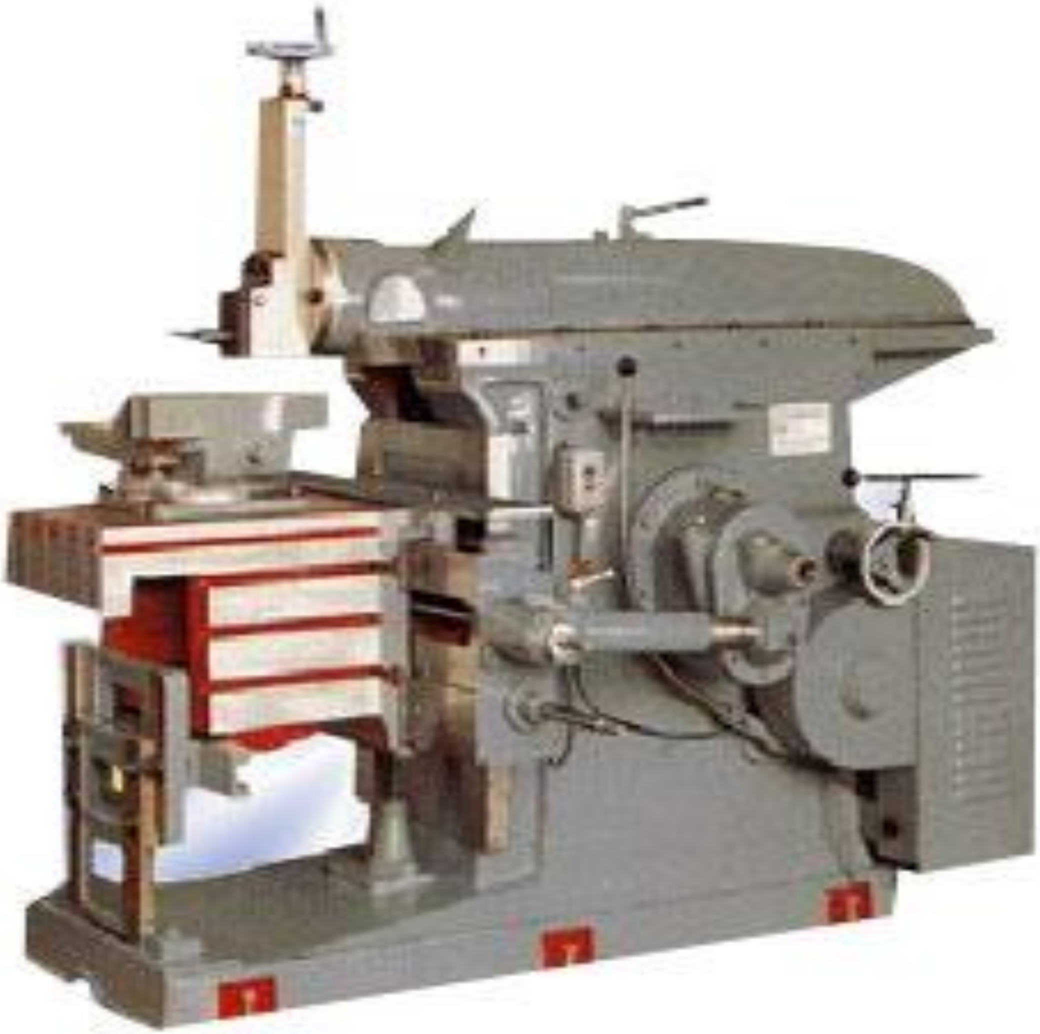


Shaper machine



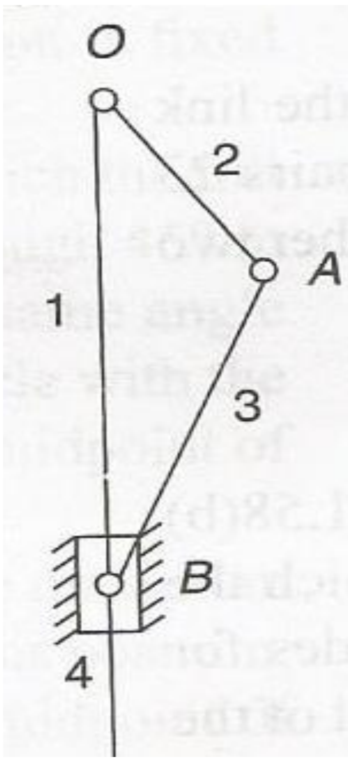


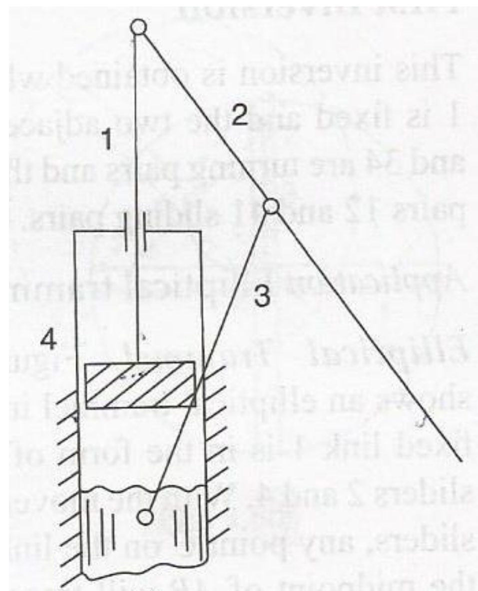
Shaper machine

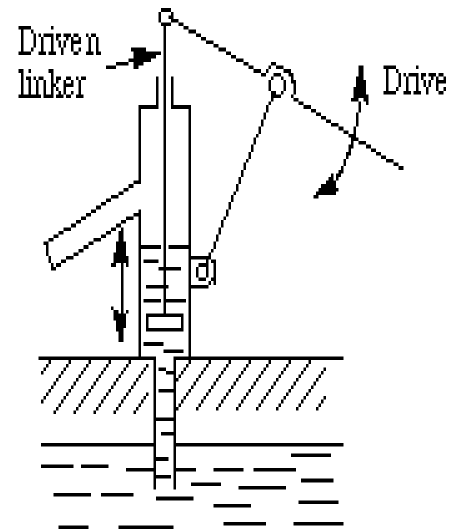


Fourth inversion

Fourth inversion is obtained by fixing link 4 of slider crank mechanism

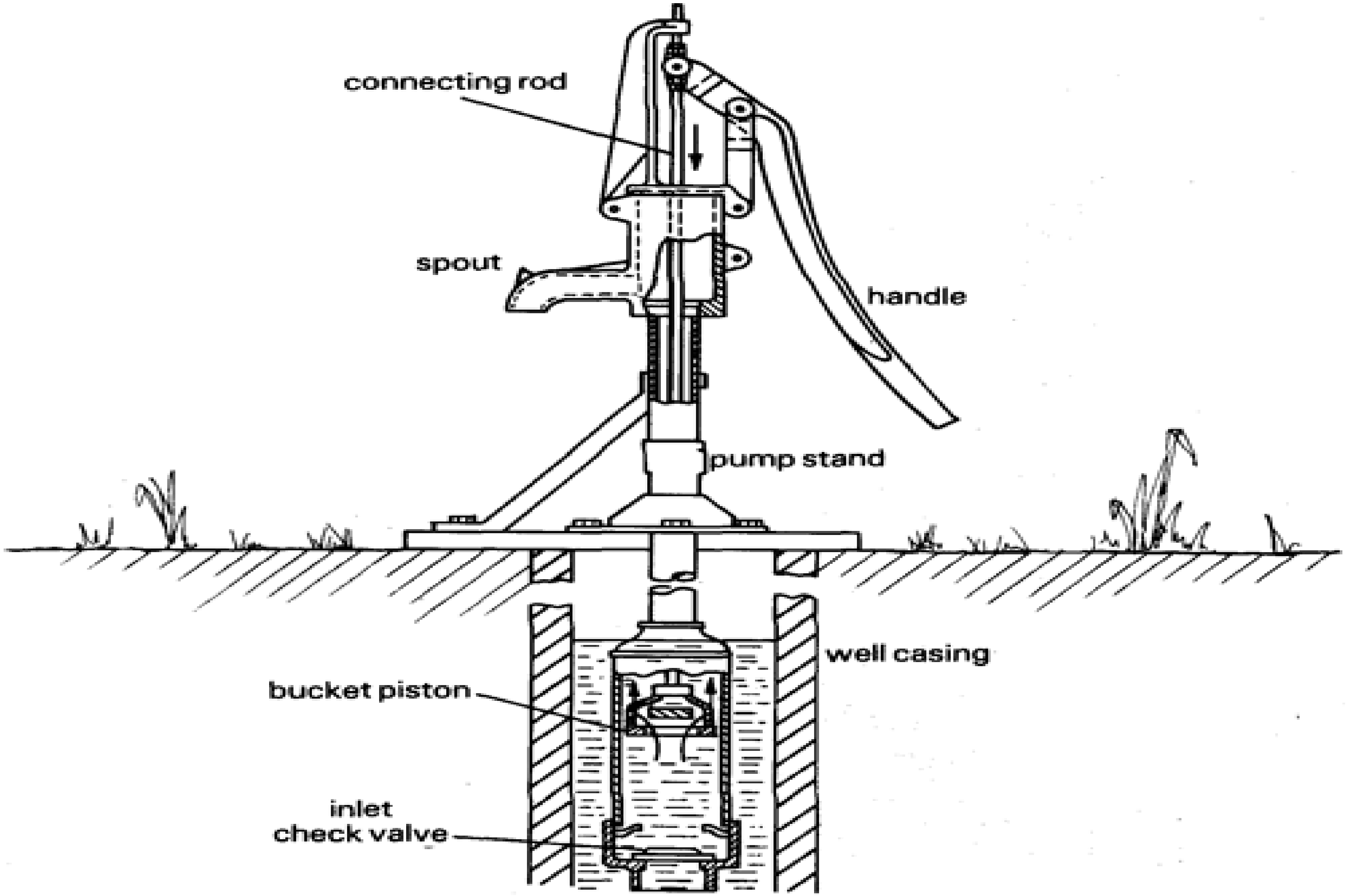






Application: Hand pump

Hand pump



connecting rod
spout
handle
pump stand
well casing
bucket piston
inlet
check valve

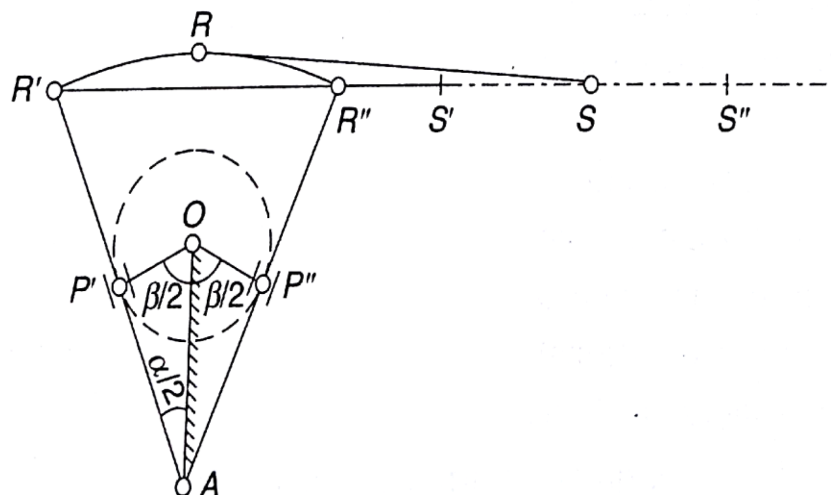
The length of the fixed link of a crank and slotted-lever mechanism is 250 mm and that of the crank is 100 mm. Determine the

(i) inclination of the slotted lever with the vertical in the extreme position,

(ii) ratio of the time of cutting stroke to the time of return stroke, and

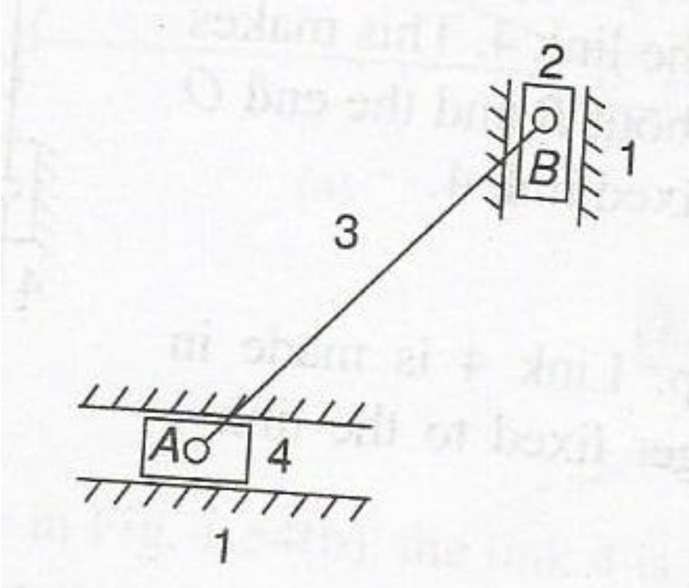
(iii) length of the stroke, if the length of the slotted lever is 450 mm and the line of stroke passes through the extreme positions of the free end of the lever.

$OA = 250$ mm; $OP' = OP'' = 100$ mm; $AR' = AR'' = AR = 450$ mm



Double slider crank mechanism

Four bar chain having two turning and sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain



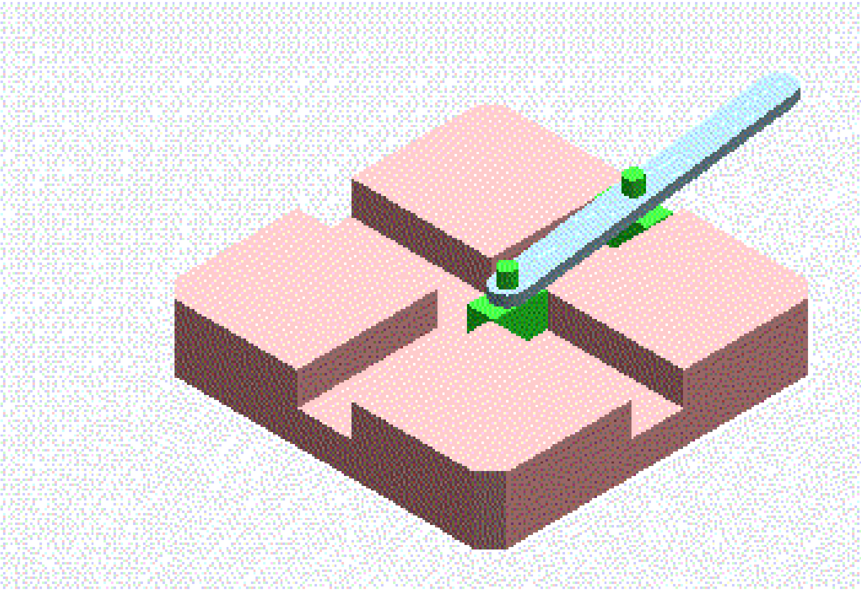
First inversion

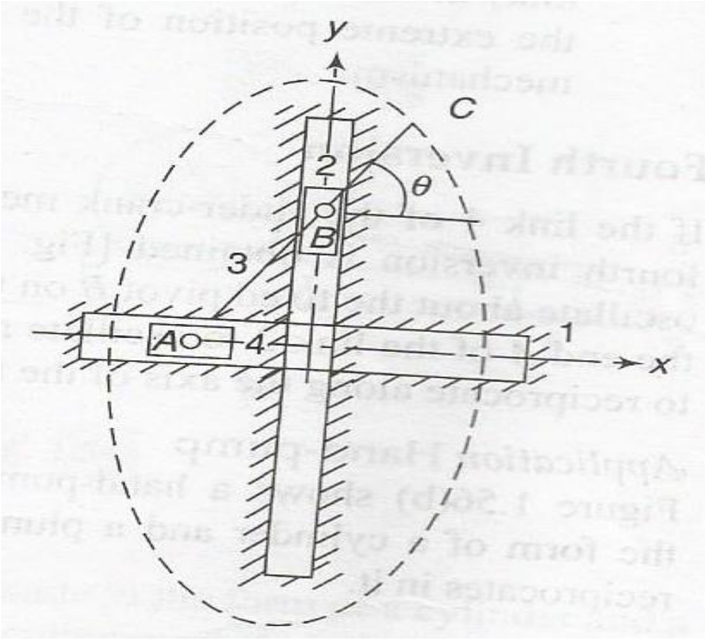
Application- Elliptical trammel

Link 1- fixed

Adjacent pairs link 2&3, link3&4– turning pairs

Adjacent pairs link 1&2, link1&4– sliding pairs





Second inversion

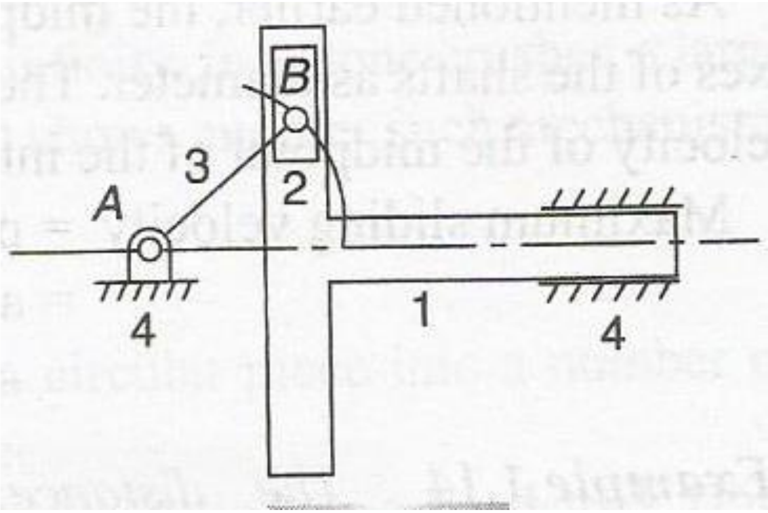
Application- Scotch yoke

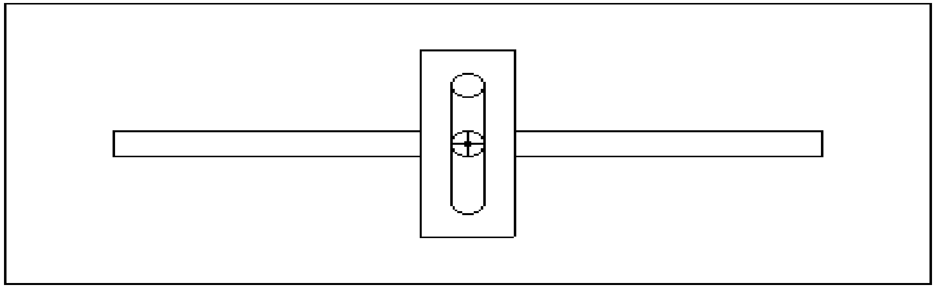
If any slide blocks of the first inversion is fixed second inversion is obtained

Scotch yoke

A scotch- yoke mechanism is used to convert the rotary motion to sliding motion. In the fig as the crank 3 rotates the horizontal portion of the link 1 slides in the fixed link 4.

It is most commonly used in control valve actuators in high pressure oil and gas pipe lines

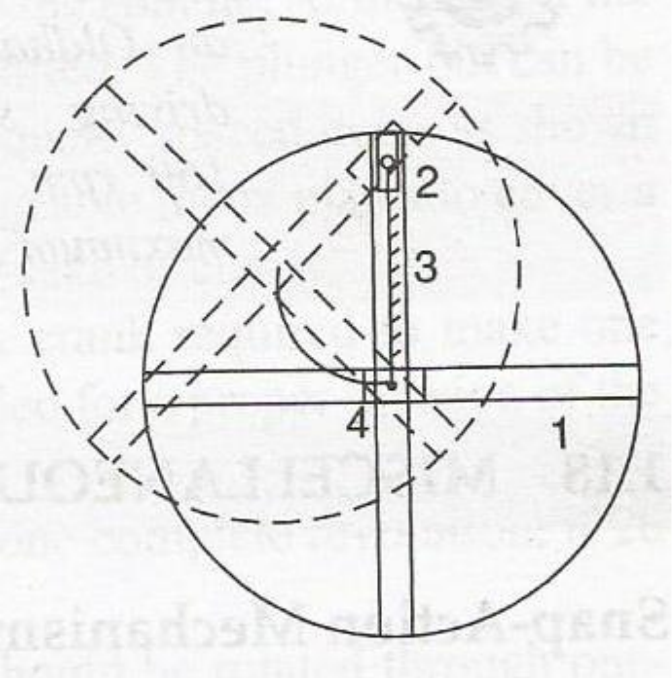




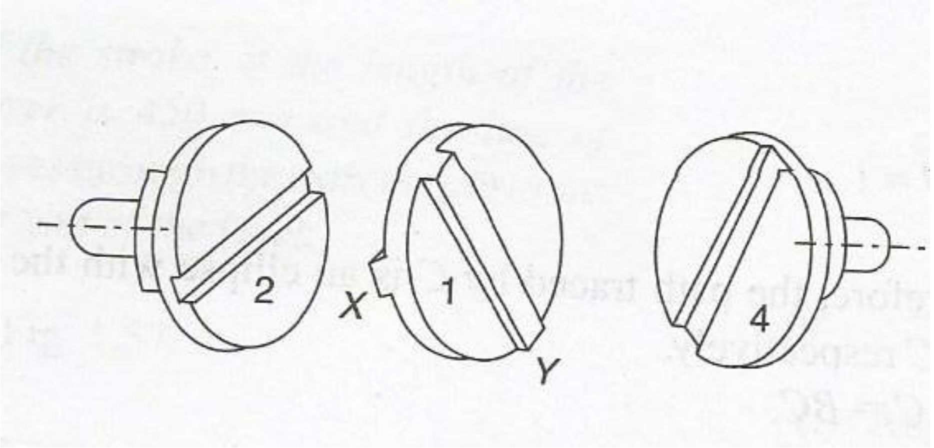
Third inversion

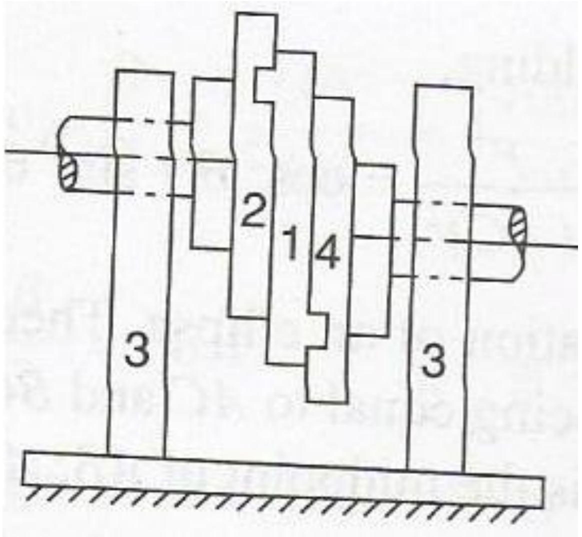
Application- Oldham's coupling

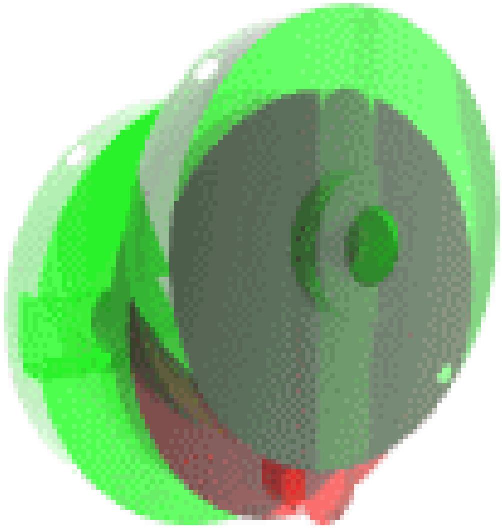
It is obtained when link 3 of the first inversion is fixed and link 1 is free to move



Oldham's coupling

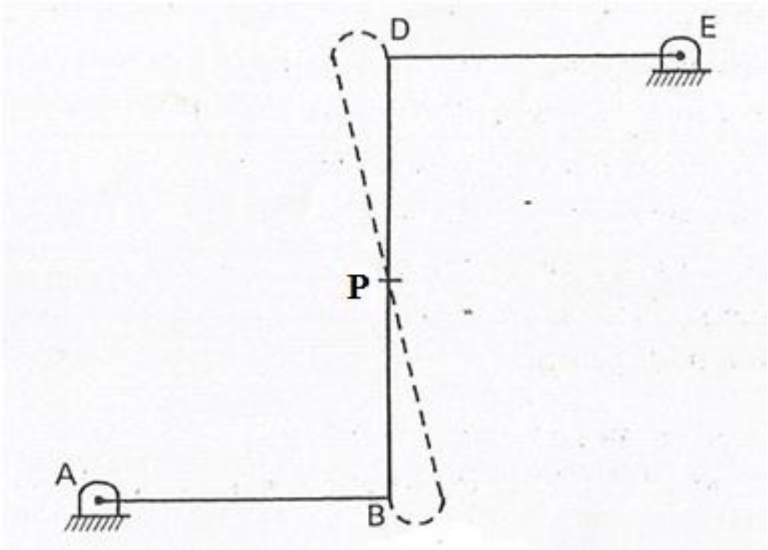


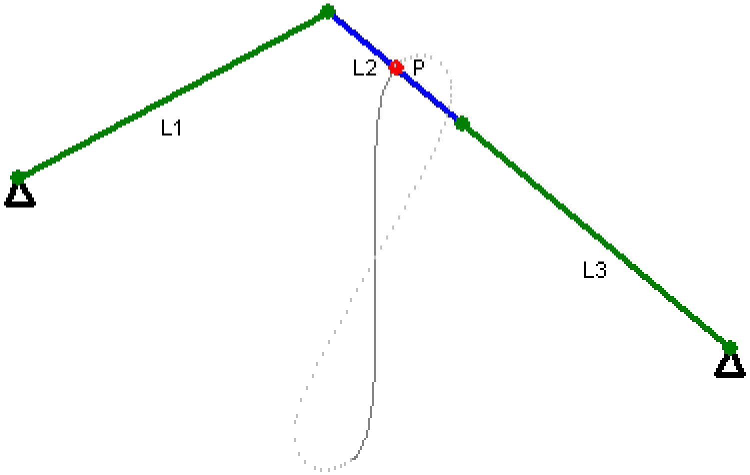




Approximate Straight line generators

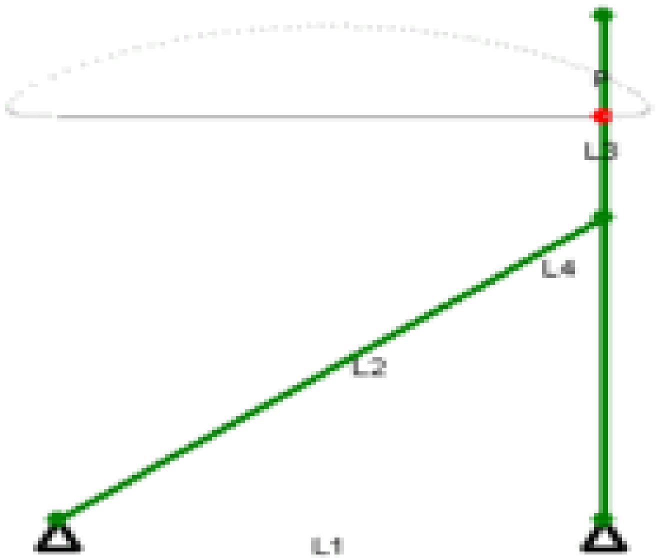
APPROXIMATE STRAIGHT LINE MECHANISM
(a) **Watt's Straight Line Mechanism (1784)**





- Link AB & DE act as levers and ends A & E are fixed.
- On small displacement of the mechanism, the tracing point P traces the shape of number **8 ,** a portion of which will be approximately straight , and hence also an example for approximate straight line mechanism

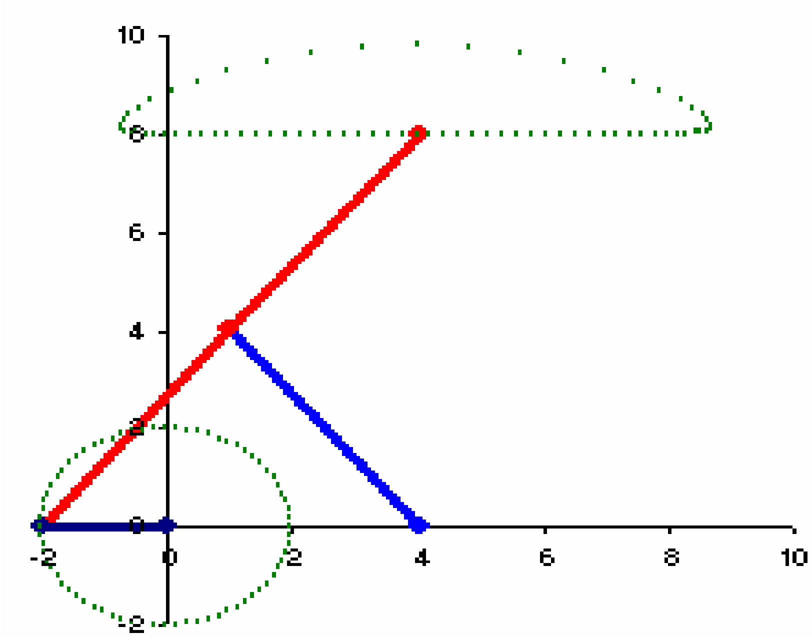
(b) Chebyshev linkage



•The **Chebyshev linkage** is a mechanical linkage that converts rotational motion to approximate straight-line motion

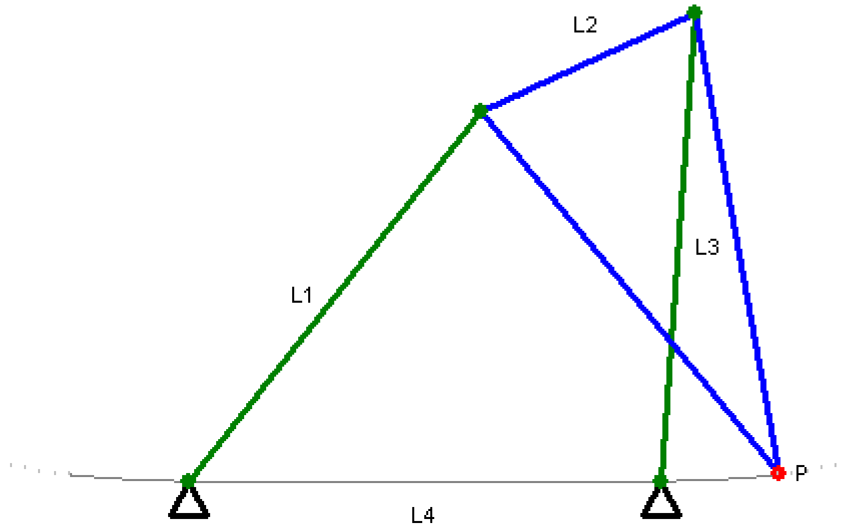
•It was invented by the 19th century mathematician Pafnuty Chebyshev who studied theoretical problems in kinematic mechanisms

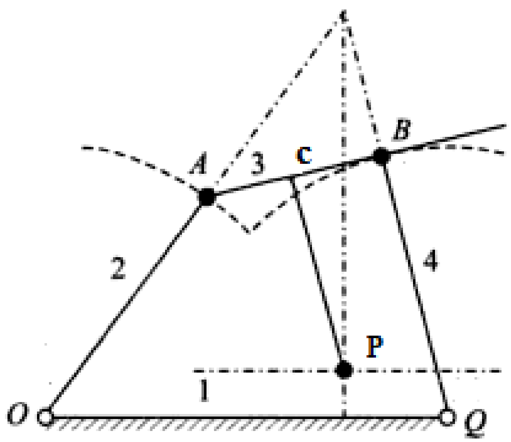
(c)Hoekens linkage



The **Hoekens linkage** is a four-bar mechanism that converts rotational motion to approximate straight-line motion with approximate constant velocity.

d) Robert’s mechanism





AO=BQ, AC=BC
CP perpendicular to AB
P is the tracing point

Accurate straight line mechanism

Condition for Exact Straight Line Motion

Referring to Fig. 1 let O be a point on the circumference of a circle of diameter OE. Let any chord *OB* is extended upto point C, such that

$$OB.OC = \text{constant}$$

Then the path of point C will be a straight line perpendicular to the diameter *OE* of the circle along the circumference of which point *B* moves. Extend *OE* to point *D* such that *CD* is perpendicular to *OD*. Join *BE*.

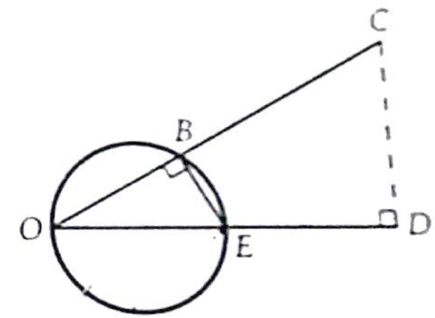


Fig 1 Straight Line Motion.

The triangles *OBE* and *OCD* are similar because angle *BOE* is common and angles *OBE* = angle *CDO* = right angle, so

$$\frac{OB}{OE} = \frac{OD}{OC}$$

or

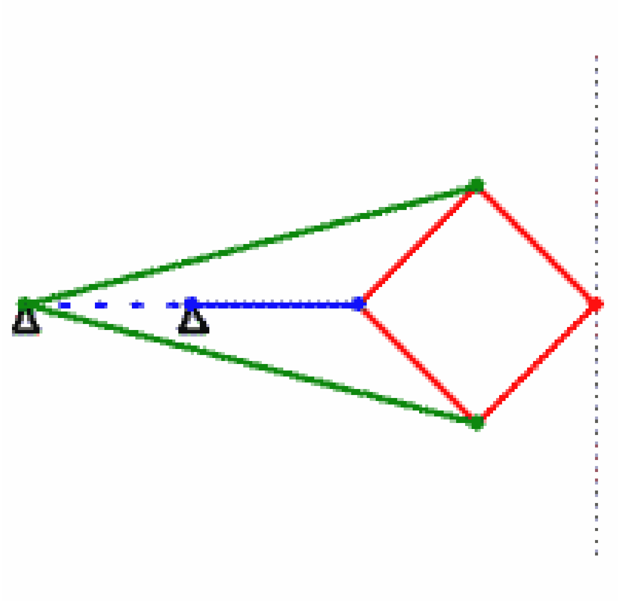
$$OD = \frac{OB}{OE}.OC$$

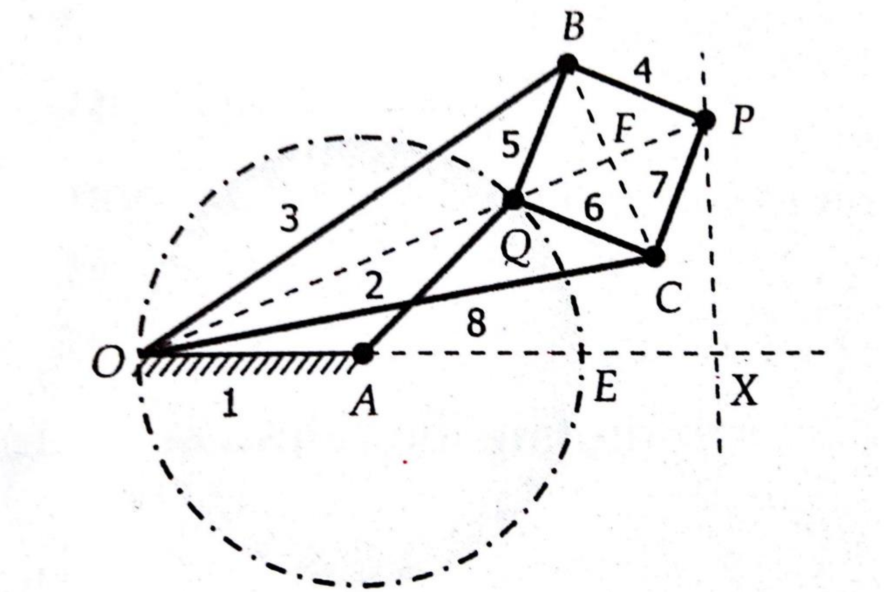
Here $OE =$ diameter of circle, constant

$OB.OC =$ constant as assumed

So *OD* will be constant as per equation (*i*). Hence, the point C moves along the straight path *CD* which is perpendicular to *OD*. Exact straight line motion mechanisms with turning pairs are described here namely Peaucellier mechanism and Hart mechanism.

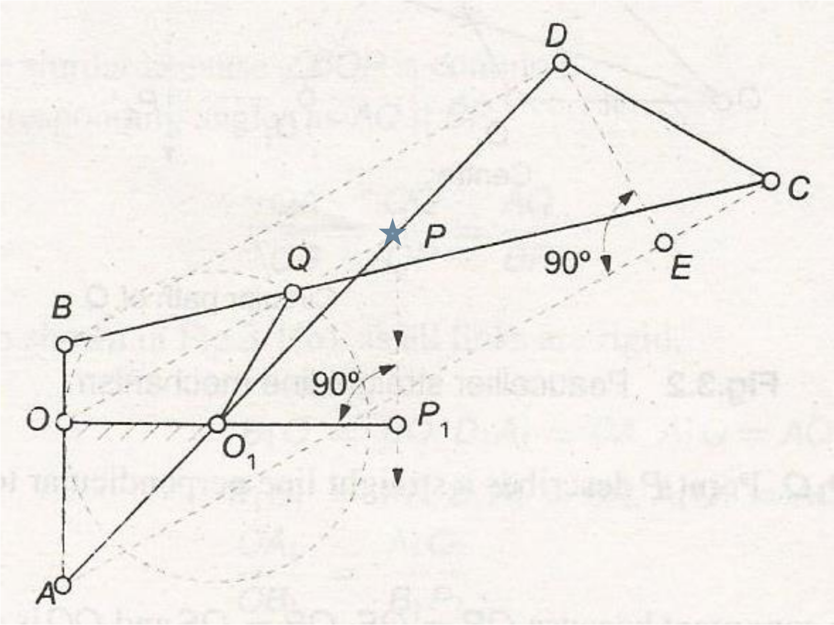
Peaucellier–Lipkin linkage





The **Peaucellier–Lipkin linkage** invented in 1864, was the first planar linkage capable of transforming rotary motion into perfect straight-line motion and vice versa

Hart Mechanism



OO_1 is the fixed link

O_1Q is the rotating link

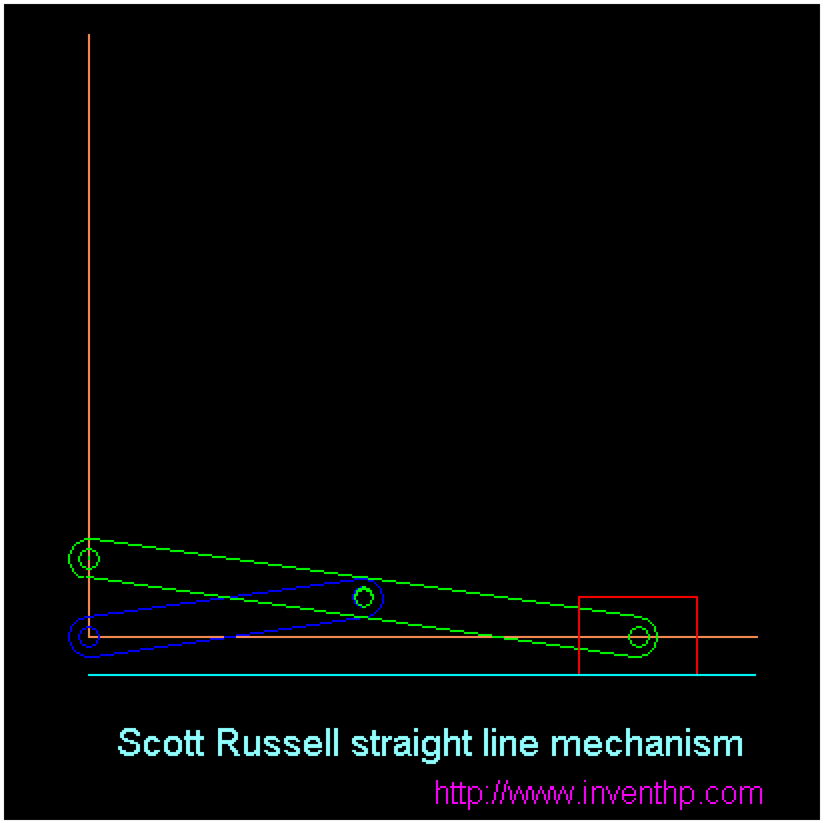
Point Q moves in a circle about O_1 and radius O_1Q

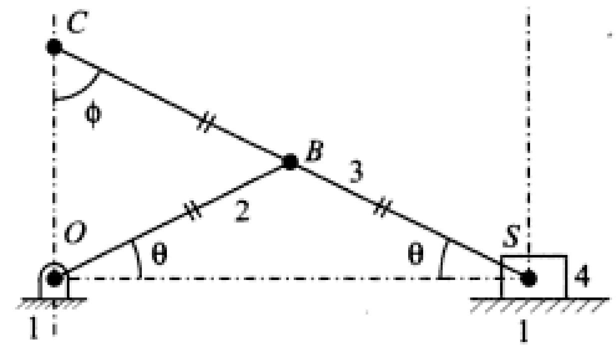
ABDC is a trapezium so that AB=CD; BD parallel to AC

BO/BA= BQ/BC=DP/DA

Point P describes the straight line perpendicular to OO_1

Scott- Russel Mechanism





OB=BS=BC

CO perpendicular to OS

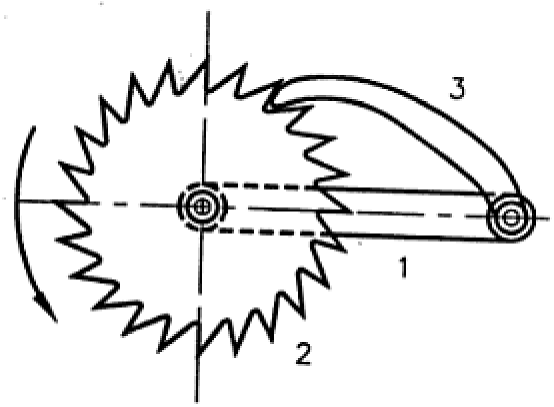
S moves in a straight line along OS

Intermittent motion mechanism

Ratchets and Escapements

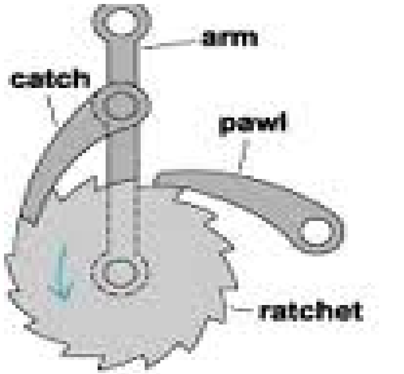
•There are different forms of ratchets and escapements used in engineering practices

•They are used in locks, clock works, jacks and many other mechanism requiring some form of intermittent motion

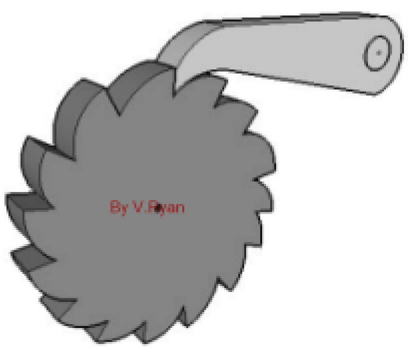


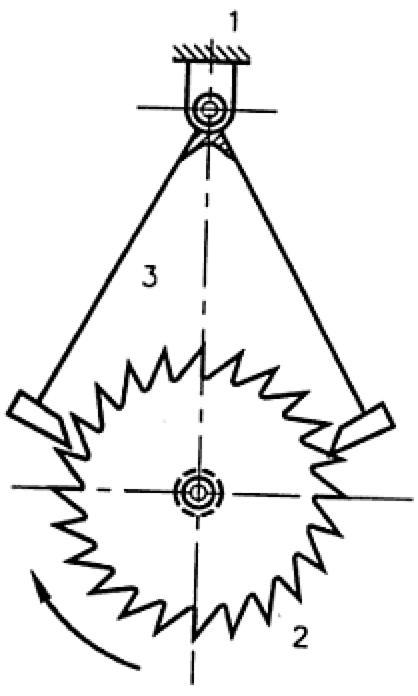
(a)

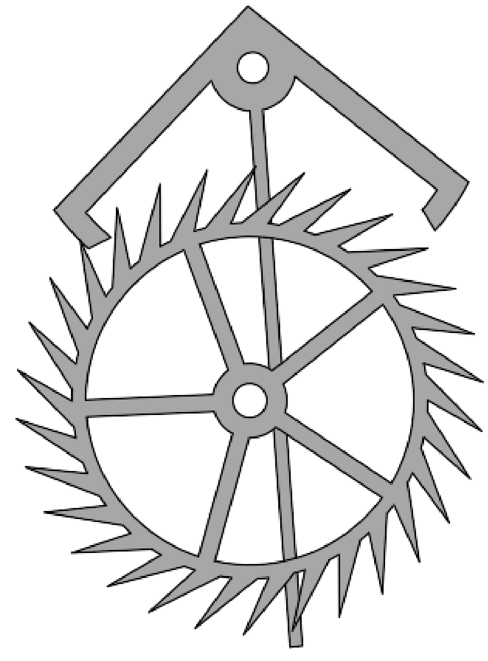
Fig (a) shows one kind of ratchets which allows the motion of the gear in one direction



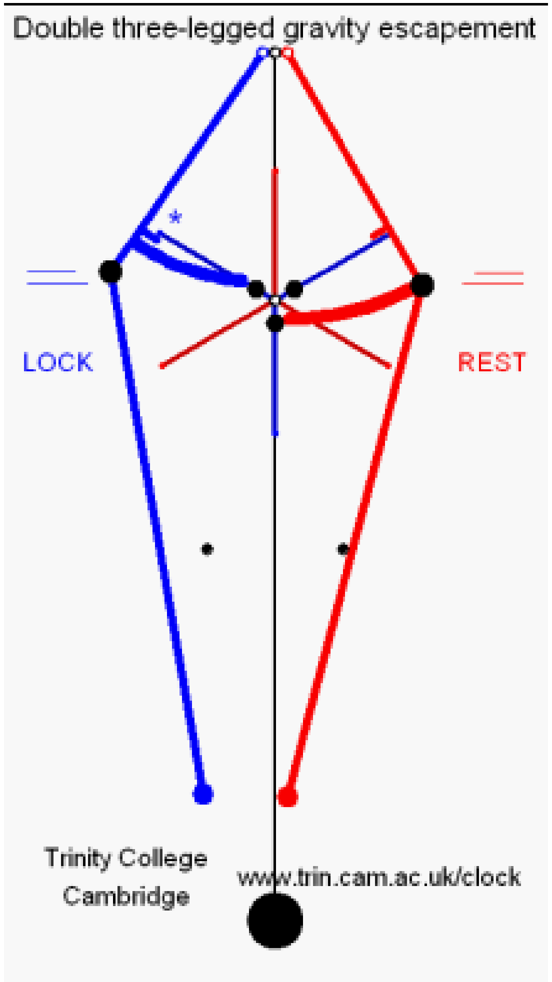
toothed ratchet





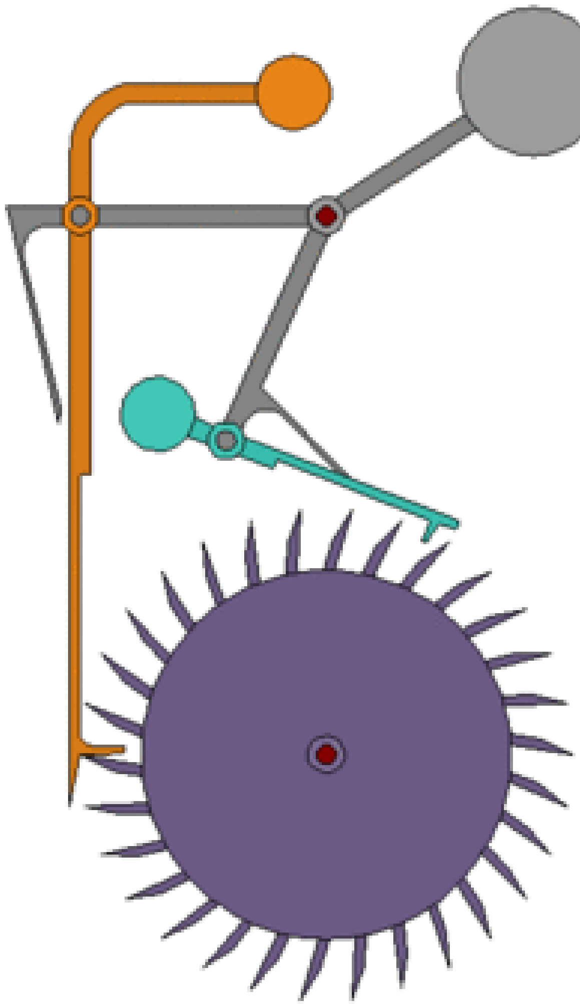


Above figure is an example for deadbeat escapement used commonly in pendulum clocks

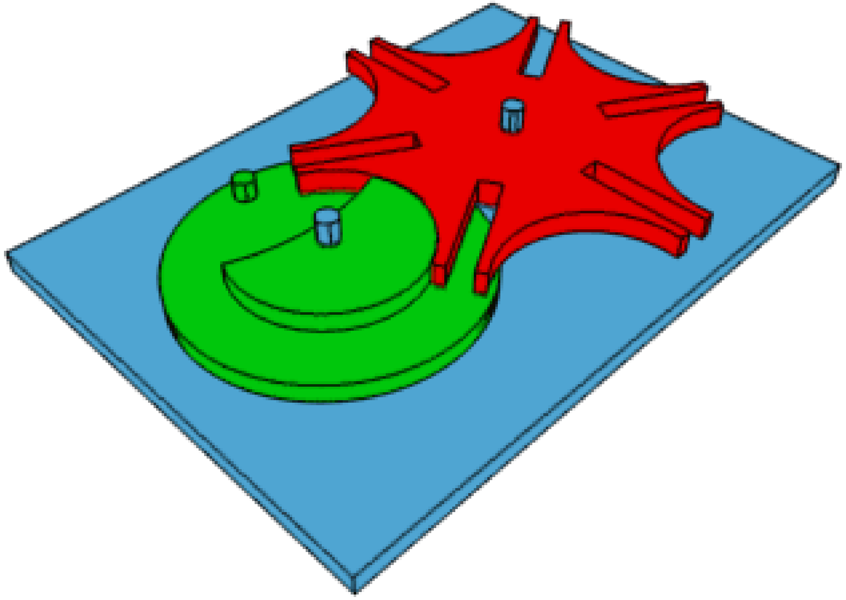


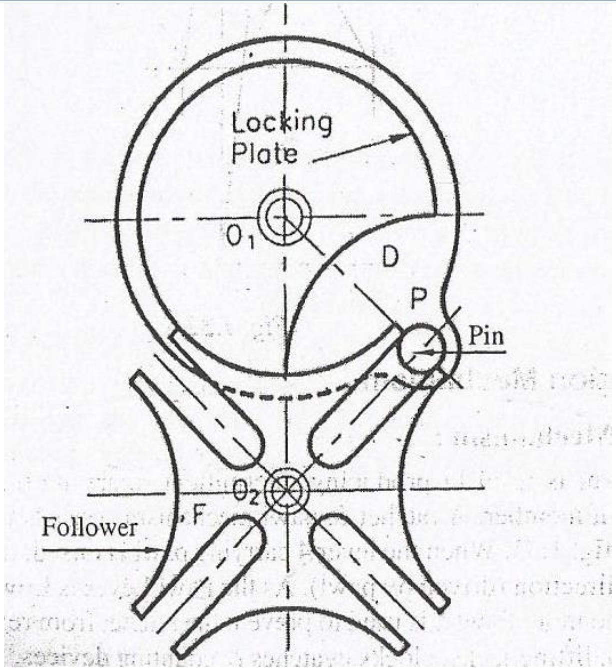
A gravity escapement uses a small weight or a weak spring to give an impulse directly to the pendulum

Grass hopper escapement



Geneva mechanism





•It is an intermittent motion mechanism

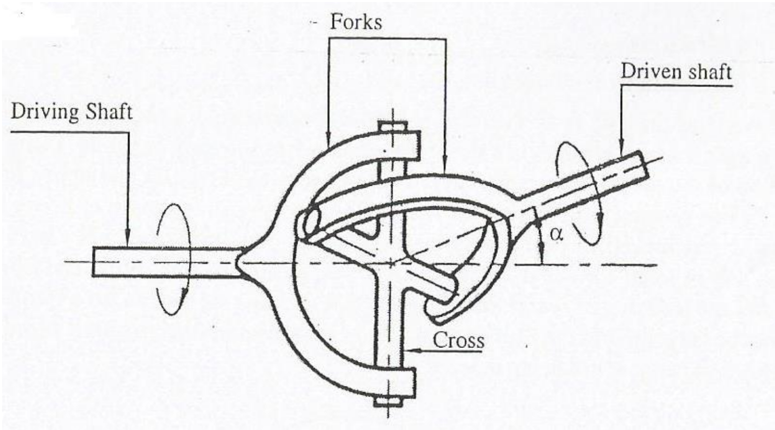
•Consists of a driving wheel D carrying a pin P which engages in a slot of the follower F

•During one quarter revolution of the driving plate, the pin and follower remain in contact and hence the follower is turned by one quarter turn

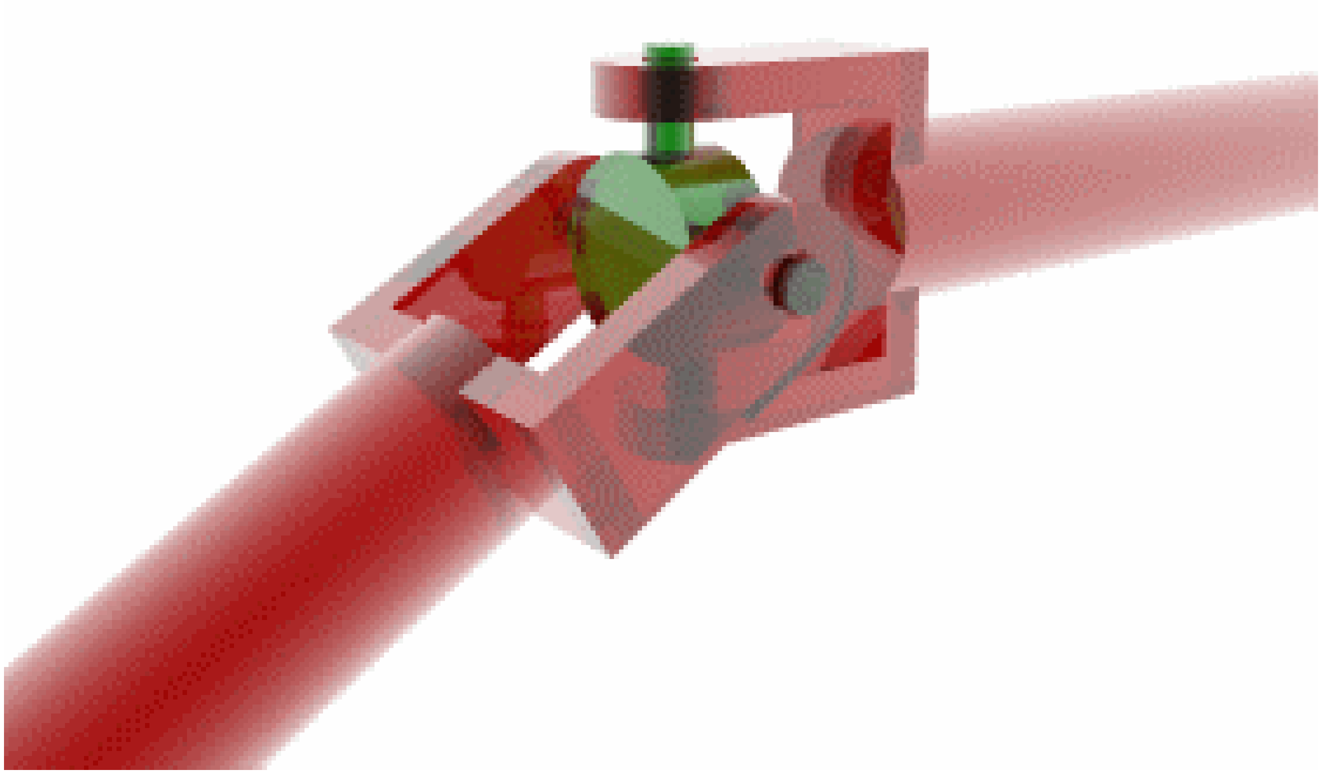
•During the remaining time of one revolution of the driver , the follower remains at rest locked in position by the circular arc

Hooke's joint or Universal coupling

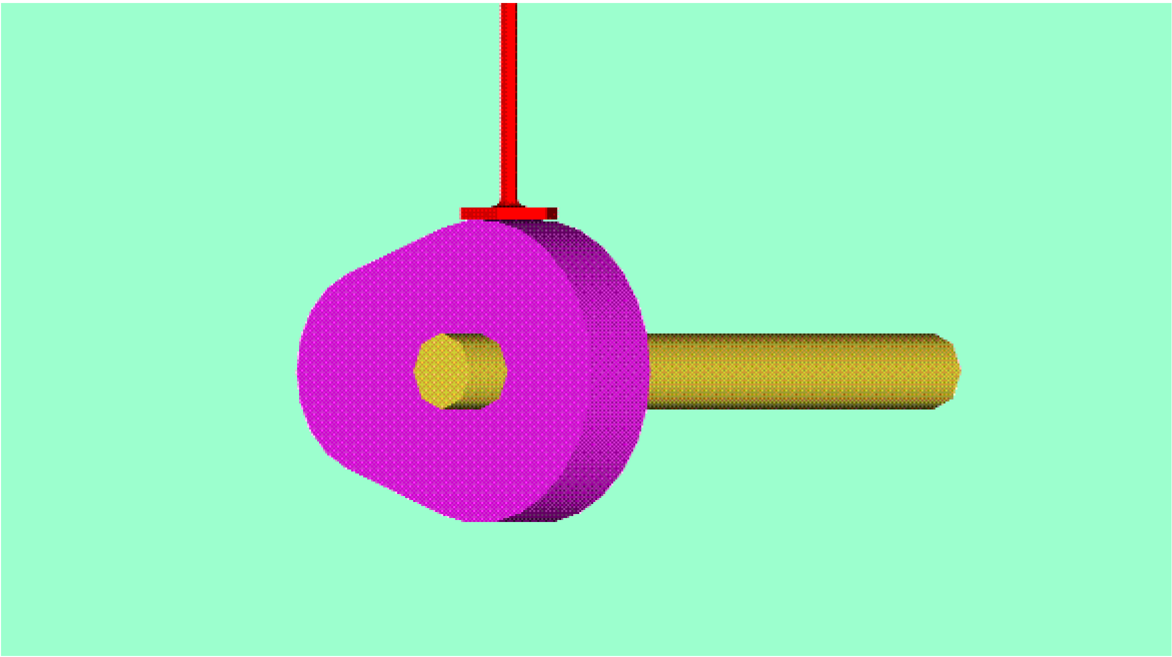
- Hooke's joint is a device that connects two shafts whose axes are neither coaxial nor parallel but intersect at a point
- Used to transmit power from the engine to the rear axle of an automobile
- Transmission of drives to different spindle in multiple drilling machine
- Knee joint in a milling machine
- Transmission of torque in rolling mills



Hooke's joint or Universal coupling

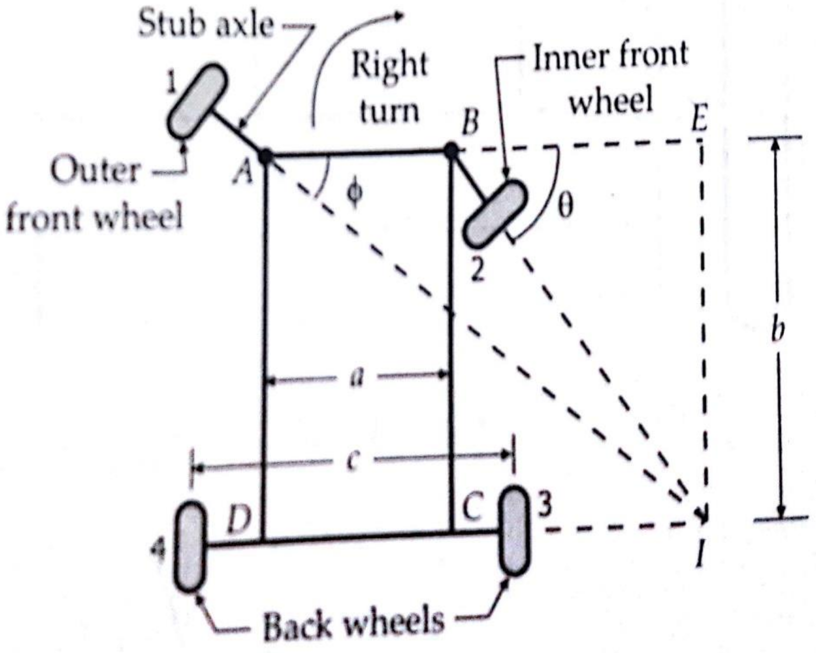


Dwell mechanism



Steering gear mechanism

Fundamental Equation of Correct Steering



Let a = Distance between pivots A and B of front axle,

b = wheel base, and

c = wheel track

From triangle BIE,

$$\cot\theta = \frac{BE}{IE}$$

and from triangle AEI,

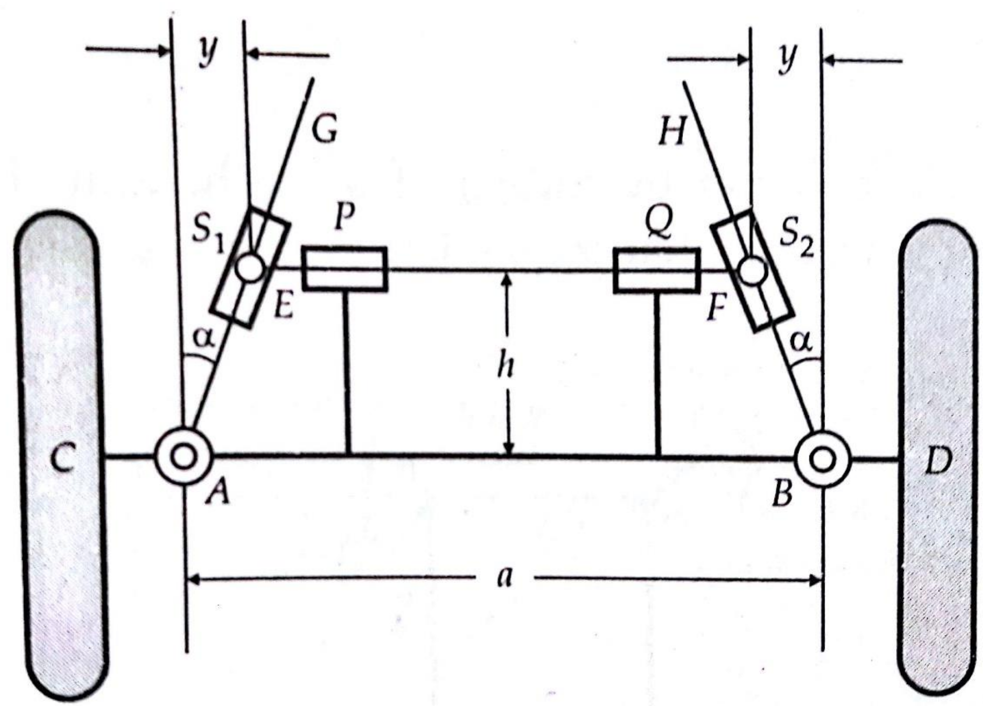
$$\cot\phi = \frac{AE}{IE} = \frac{AB}{IE} + \frac{BE}{IE} = \frac{a}{b} + \cot\theta$$

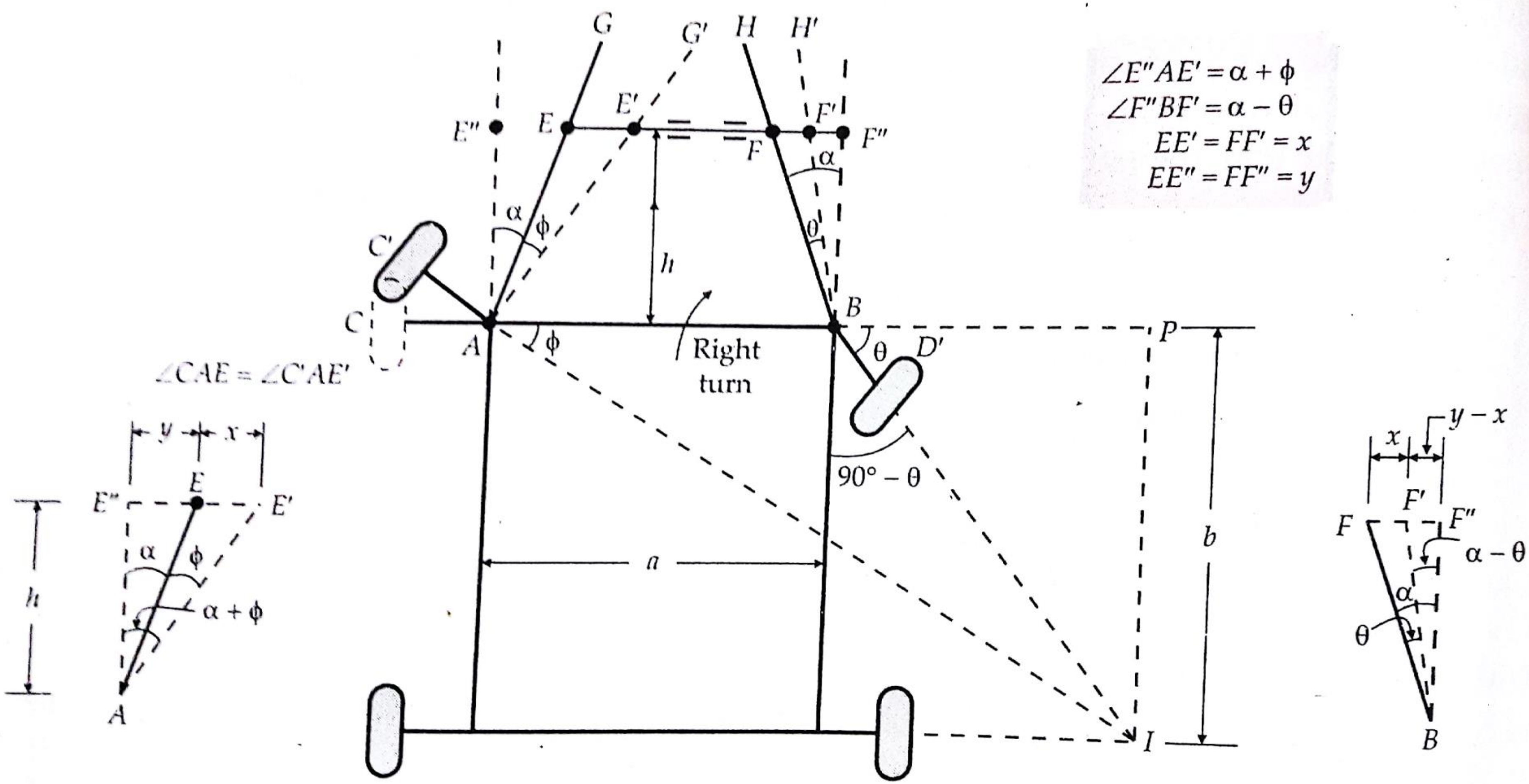
$$\cot\phi - \cot\theta = \frac{a}{b}$$

Types of Steering Gear

- Davis steering gear (which has sliding pairs)
- Ackermann steering gear (which has turning pairs)

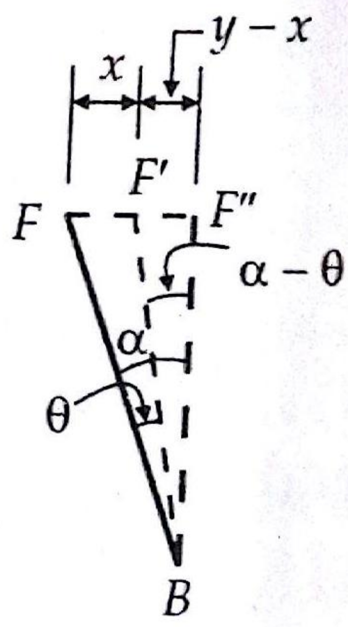
Davis steering gear mechanism





G
G′
H
H′
E″
E
E′
F
F′
F″
α
ϕ
θ
C′
C
h
A
B
P
D′
Right
turn
∠CAE = ∠C′AE′
90° − θ
a
b
I
∠E″AE′ = α + ϕ
∠F″BF′ = α − θ
EE′ = FF′ = x
EE″ = FF″ = y
y
x
α + ϕ
y − x
α − θ

From triangle F' F'' B



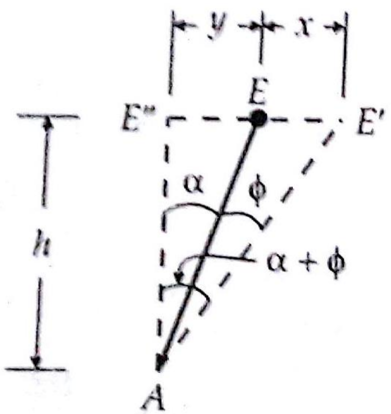
$$\tan(\alpha-\theta)=\frac{F'F''}{F''B}=\frac{FF''-FF'}{F''B}$$

$$\frac{\tan\alpha-\tan\theta}{1+\tan\alpha\,.\,\tan\theta}=\frac{y-x}{h}\quad\text{and}\quad\tan\alpha=\frac{y}{h}=\frac{EE''}{AE''}=\frac{FF''}{BF''}$$

$$\frac{\frac{y}{h}-\tan\theta}{1+\frac{y}{h}\,.\,\tan\theta}=\frac{y-x}{h}$$

$$\frac{y-h\tan\theta}{h+y\,.\,\tan\theta}=\frac{y-x}{h}$$

$$\tan\theta=\frac{hx}{y^2-xy+h^2}$$ Eq. (1)



From triangle E'' E' A

$$\tan(\alpha + \phi) = \frac{E''\,E'}{AE''} = \frac{E''\,E + EE'}{AE''}$$

$$\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \,.\, \tan\phi} = \frac{y + x}{h}$$

Solving it, we can write,

$$\tan\phi = \frac{hx}{y^2 + xy + h^2} \qquad \text{Eq. (2)}$$

But for correcting steering, $\cot\phi - \cot\theta = a/b$

From (*ii*) and (*i*),

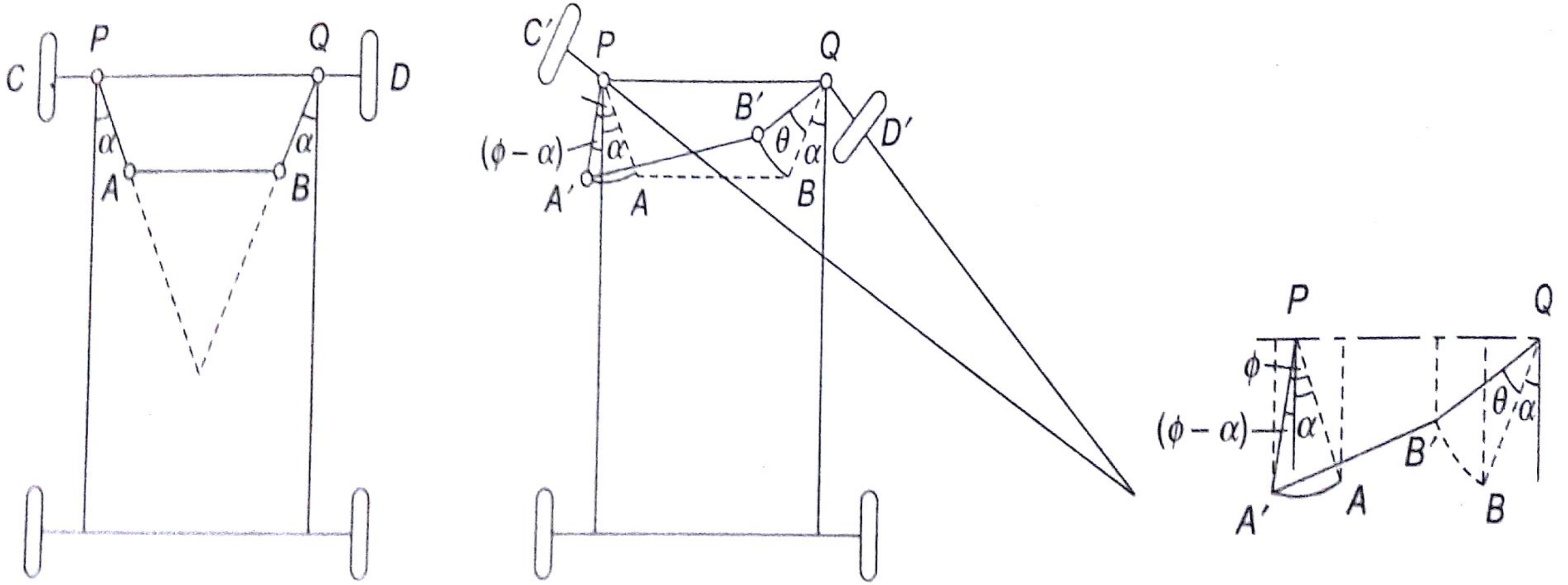
$$\cot\phi - \cot\theta = \frac{y^2 + xy + h^2 - y^2 + xy - h^2}{hx}$$

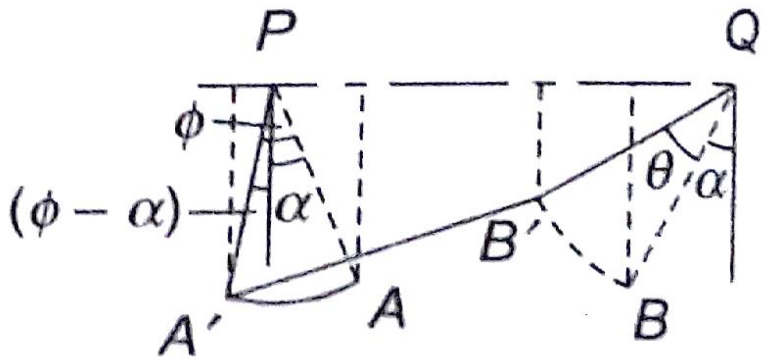
$$\frac{a}{b} = \frac{2xy}{hx} = \frac{2y}{h}$$

$$\frac{y}{h} = \frac{a}{2b} \text{ or } \tan\alpha = a/2b$$

The value of α is generally 11° to 14° and $\frac{a}{b} = 0.4$ to 0.5.

Ackermann steering gear





$$\text{Projection of } BB' \text{ on } PQ = \text{Projection of } AA' \text{ on } PQ$$

$$QB\,[\sin(\alpha + \theta) - \sin\alpha] = PA\,[\sin\alpha + \sin(\varphi - \alpha)]$$

or
$$\sin(\alpha + \theta) - \sin\alpha = \sin\alpha + \sin(\varphi - \alpha) \qquad (PA = QB)$$

$$(\sin\alpha\cos\theta + \cos\alpha\sin\theta) - \sin\alpha = \sin\alpha + \sin\varphi\cos\alpha - \cos\varphi\sin\alpha$$

$$\sin\alpha\,(\cos\theta + \cos\varphi - 2) = \cos\alpha\,(\sin\varphi - \sin\theta)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\varphi - \sin\theta}{\cos\theta + \cos\varphi - 2}$$

$$\tan\alpha = \frac{\sin\varphi - \sin\theta}{\cos\theta + \cos\varphi - 2}$$

where θ and φ are the values of angles for the correct gearing.